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CONTRIBUTION TO REVIEW OF THE EFFECTS LEKHNITSKII INFLUENCE TO DYNAMIC PARAMETERS OF COMPOSITE STRUCTURES

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Dynamic analysis of anisotropic (in this paper generalized orthotropic) structure is often reduced to the analysis of quasi isotropic structures. In this way, it is possible to obtain results of acceptable accuracy. Improvement of results can be achieved primarily through the appropriate stiffness matrix, which includes specifics of composite (anisotropic) material concerning the interactions of normal stress and shear strain, apropos shear stress and dilatation (based on the effects of Leknitskii).

Results of the analysis for the two above-mentioned models of a composite structure (generally orthotropic and quasi-orthotropic), are presented in graphical and tabular form.

Key words: Dynamic analysis, Composite materials, Lekhnitskii effects

INTRODUCTION

Structural analysis will be based on the theoretical concept of composites in accordance with [1]. It is evident that the concept of structural analysis of isotropic structures can not be applied, therefore, through the model matrix of elasticity and stiffness of orthotropic structures, the specific characteristics of the considered structures will be entered, as it normally done when calculating the composite structure.

This paper is prepared and presented based on the methodological developments of different authors [8,9,10] and basically is a rational attempt to be reasonable contribution to the R& D of interest for the design of structures on the basis of modern composite materials with use of computers. [6,7,9].

Here will be carried out analysis of the impact of some of the most significant effects that are not necessarily seen in the spatial structure based on different materials.

Determination of natural frequencies and eigen modes of oscillation is reduced to the solution of homogeneous differential equations of motion. In principle there are two possible cases:

- determination of dynamic parameters of the model without damping;
- determination of dynamic parameters of the model with the damping [2].

Simpler is the first case and it will be here analyzed (using the finite element method - FEM), with a full emphasis on specifics loaded through the stiffness matrix for plate anisotropic structures based on the interaction between normal stress and shear strain and shear stress and dilatation respectively [1,3]. Specifically, all members of the matrix of matherial stiffness or elasticity $\overline{Q}$ - and flexible $\overline{S}$ - would be different from zero, equations (01) and (02).

$$\overline{Q} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{21} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{61} & \overline{Q}_{62} & \overline{Q}_{66} \end{bmatrix}; \overline{Q}_{ij} \neq 0 \ (i = 1,2,6; j = 1,2,6) \quad (1)$$

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$$\overline{S} = \overline{Q}^{-1} = \begin{bmatrix} \overline{S}_{11} & \overline{S}_{12} & \overline{S}_{16} \\ \overline{S}_{21} & \overline{S}_{22} & \overline{S}_{26} \\ \overline{S}_{61} & \overline{S}_{62} & \overline{S}_{66} \end{bmatrix}; \overline{S}_{ij} \neq 0 \ \left(i = 1,2,6; j = 1,2,6\right) \quad (2)$$

Comparing the results of the analysis of a model for the various concepts of dynamic analysis will be discussed under chapter Results analysis dynamic parameters

BASIC THEORETICAL RELATION

Set problem is defined by differential equations, (03) to (08), according to [2],

$$M\ddot{\delta} + K\delta = 0 \quad (3)$$

For the vector of node displacements,

$$\delta = \overline{\delta}e^{i\omega t} \quad (4)$$

the problem is reduced to,

$$\left(-\omega^2 M + K\right)\overline{\delta}e^{i\omega t} = 0 \quad (5)$$

respectively, for the determination of non-trivial solutions need to be a determinant of this system is equal to zero,

$$\left|-\omega^2 M + K\right| = 0 \quad (6)$$

where M and K the global stiffness matrix and matrix of inertia are obtained on the basis of elementary matrices,

$$m_{(k)} = \int_V N^T \rho N dV \quad (7)$$

-matrix of inertia of elements.

$$k_{(k)} = \int_V B^T \overline{Q} B dV \quad (8)$$

-matrix of stiffness of elements.

Expressions (07) and (08) for elements with isotropic material are well known from the literature [2]. Closer will be discussed only matrix that brings the substantive changes to the structural concept of elements.
Geometric concept of composite plate is the same with isotropic element.

Matrix has been adopted for the concept of symmetric composite plates with respect to the middle plane of elements [1,5], equation (09).

$$\overline{Q} = T^{-T} Q T \quad (9)$$

where

: - Q - basic orthotropic elasticity matrix of material (E1, E2, ν12, G12), in which the geometric-xy and material-12 coordinate system coincide;

- T - transformation matrix, the expression (10), where xy and 12 do not coincide (θ - angle of transformation),), Figure 1.

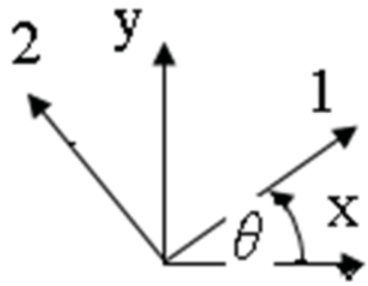

Figure 1. Geometric-xy and material-12 coordinate systems

$$T = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \quad (10)$$

Based on the eq. (09) and (10), comes to the relation (11) and (12), according to [1].

$$S_{16} = S_{61} = \frac{\eta_{xy,x}}{E_x} = \left(\frac{2}{E_i} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin\theta\cos^3\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin^3\theta\cos\theta \quad (11)$$

$$S_{26} = S_{62} = \frac{\eta_{xy,y}}{E_y} = \left(\frac{2}{E_i} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin^3\theta\cos\theta - \left(\frac{2}{E_2} + \frac{2\nu_{12}}{E_1} - \frac{1}{G_{12}}\right)\sin\theta\cos^3\theta \quad (12)$$

where: $\eta_{ij,\alpha}$ i $\eta_{i,\alpha\beta}$ $(\eta_{ij,\alpha}/E_\alpha = \eta_{i,\alpha\beta}/G_{\alpha\beta})$

- coefficient of mutual influence of the first and second row, known as the coefficients of Leknitskii. In principle, functional dependence $\eta(\theta)$ is shown in Figure 2.

It can be concluded that, in describing the model with orthotropic or quasi-orthotropic structure, the greater difference in the accuracy of the analysis results is if we have larger absolute values of coefficients η.

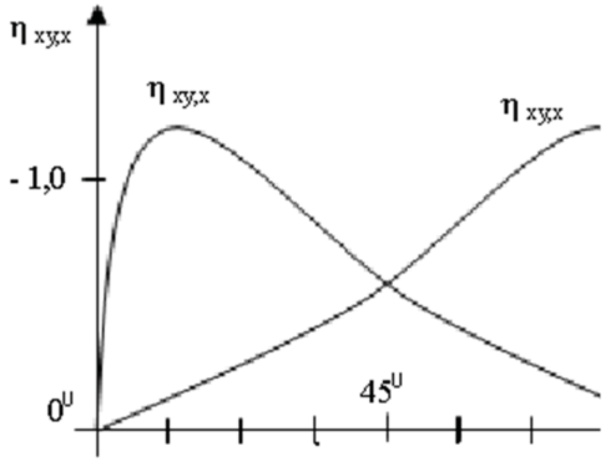

Figure 2. Functional dependency . η(θ)

STRUCTURAL ANALYSIS OF THE MODEL

Analysis of the basic dynamic parameters, based on the above mentioned equations, will be carried out for model shown in Figure 3.

Now, for the next calculation and comparing results, for the model of quasi-orthotropic structure, the concept (13), and for the model with generalized orthotropic structure, the concept (01), are adopted.

$$Q = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \quad (13)$$

t = 10 – plate thickness
b = 100– length of finite element cathetus "Δ"
E1 = 10 000 daN/mm2, E2 = 1 000 daN/mm2
ν_{12} = 0,3, G12 = 500 daN/mm2, θ = 45^0,
ρ = 2,4 kg/dm3

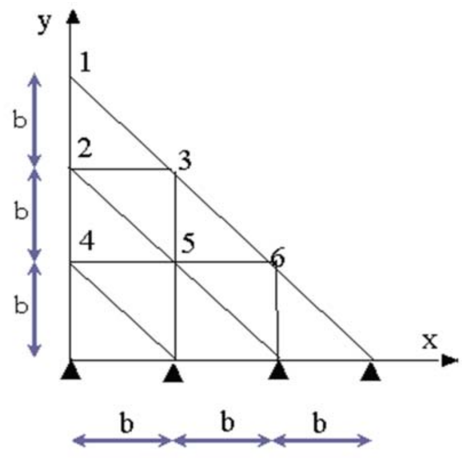

Figure 3. Structural model

Inertia matrix and stiffness matrix of elements, as well as the matrix of inertia and stiffness of the system, defined by the foregoing equations and theoretical relations set out in [2].

RESULTS ANALYSIS DYNAMIC PARAMETERS

Modeling, calculation and the results were carried out on computers, the application of FEM to define dynamic parameters and the modal matrix of a graphic interpretation of its own modes of oscillation is done by applying generally accepted MATLAB 6.5 package.

* Eigen value of frequency [s]

- The concept quasi-orthotropic structure:

ω_1= 205,ω_2 = 375,ω_3 = 556,ω_4 = 669,
ω_5 = 950;

- The concept generalized orthotropic structure:
ω_1 = 131,ω_2 = 223, ω_3 = 334, ω_4 = 433,
ω_5 = 573.

The relative error is very pronounced, for the adopted base material is even greater than 50%. Values of ωs are in more accurate for generalized orthotropic concept (second concept).

* Eigen value of oscillation modes

Eigen value of oscillation modes are determined by [2]. It is evident that modes of oscillation of the discussed concepts in the appropriate measure are different. The largest discrepancies are present in modes III and IV.

Relative displacements (without the written of displacement of node 5) in relation to their eigenvalue perception of the oscillation modes are shown in the form of modal matrix μ' i μ'':

So, for two different concepts of where the budget for the concept (15) deliberately entered error to show the obvious way, as are differences, even very important, may appear and give the wrong picture about the dynamic behavior of real structures (14) should be she descriptions inadequate mechanical concept.

All this is further reflected at the level of graphical display modes, the differences are evident. Graphical display modes I and III is shown in Figure 4. Of course, possible jegraficki show and all the other modes, but here the emphasis is primarily only on the qualitative perception of the difference in the budget for the exact and approximate structure model, which is also shown.

- for generalized orthotropic structure

$$\mu' = \begin{bmatrix} -1.0 & 1.0 & -1.0 & -0.51 & 1.0 \\ 0.46 & 1.0 & 0.92 & 1.0 & -0.89 \\ -0.73 & 0.42 & -0.21 & 0.29 & -0.19 \\ 0.36 & 0.95 & 0.28 & -0.15 & 0.14 \\ -0.69 & 0.42 & -0.06 & -0.17 & -0.43 \\ 0.47 & 0.35 & 0.20 & 0.37 & 0.38 \\ -0.34 & 0.03 & 0.40 & 0.74 & 0.25 \\ 0.16 & 0.62 & -0.25 & -0.58 & -0.24 \\ -0.26 & 0.03 & 0.51 & -0.52 & 0.40 \\ 0.23 & 0.03 & -0.39 & 0.60 & -0.36 \end{bmatrix} \quad (14)$$

- for quasi-orthotropic structure

$$\mu'' = \begin{bmatrix} -1.0 & 0.2 & -1.0 & 0.7 & 1.0 \\ 0.0 & 1.0 & 0.0 & 1.0 & -0.3 \\ -0.6 & -0.2 & 0.0 & -0.4 & -0.3 \\ -0.2 & 0.5 & -0.1 & 0.4 & -0.1 \\ -0.6 & 0.2 & 0.2 & -0.5 & -0.5 \\ 0.1 & 0.4 & 0.3 & -0.1 & 0.2 \\ -0.2 & -0.1 & 0.5 & 0.1 & -0.1 \\ 0.2 & 0.3 & 0.2 & -0.2 & 0.0 \\ -0.2 & -0.4 & 0.3 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.2 & -0.3 & 0.6 \end{bmatrix} \quad (15)$$

Graphical display of modes I and III is given in Figure 4.

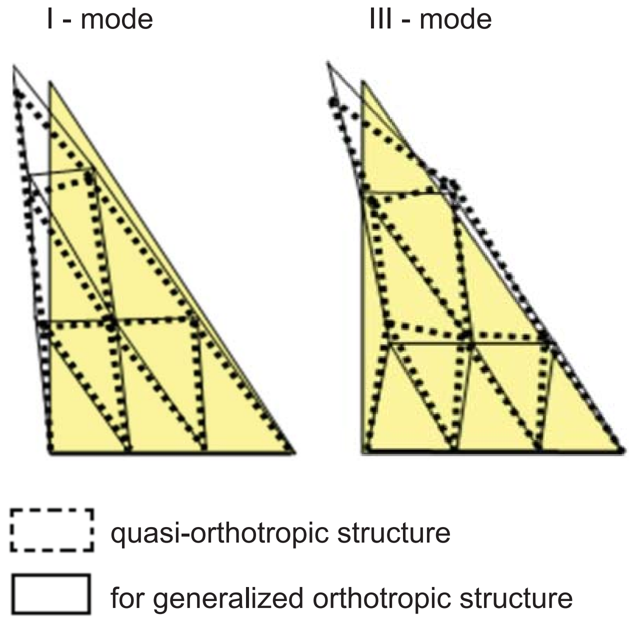

Figure 4. Graphical display of modes I and III

Inconsistency results from the dynamic analysis of the concepts outlined generalized orthotropic structures, shows that the correct calculation must include all specific of materials expressed by the mechanical characteristics of the stiffness matrix.

Graphic, as very indicative, are presented modes I and III. Given the modal matrix with clearly distinct zones in which a significant discrepancy of the real and approximate concept are appeared as evident result.

CONCLUSIONS

In the introduction, it was pointed out that the dynamic analysis of anisotropic (in this paper generalized orthotropic) structure is often reduced to the analysis of quasi-isotropic structure and, thus, possible to obtain results of acceptable accuracy. In further consideration is proved, that the improvement in results can be achieved primarily by application of appropriate stiffness matrix, in which they included specific composite (anisotropic) material concerning the interaction of normal stresses and strains smičućnih or shear stresses and dilatation (based on the effects Lekhnitskog).

The findings of two relevant model of a composite structure (generalized orthotropic and quasi-orthotropic), are presented in graphical and tabular form. Inconsistency results indicated generalized concepts of dynamic analysis of orthotropic structures, shows that the correct

Budget must include all specific materials expressed by the mechanical properties of the matrix stiffness. Graphically, as very indicative, show a numerical modes and modal matrix with clearly identified areas where significant discrepancy is evident in the results the real and the approximate concept.

Compatibility analysis would relate to aspects of viscoelasticity and viscoplasticity of composite structures for what will be focused our further work.

REFERENCES

1) Robert M. J.:, “Mechanics of Composite Materials”: Scripta Book Company, Washington, D.C. McGraw-Hill Internaional Book Company, 1975, pp. 355.

2) Sekulović M.: „Finite elements“, Construction book (Građevinska knjiga), Belgrade, 1984

3) Tomic R.: „Optimisation of composite structure for an engineering model with regard to fiber orientation“, Conference “STRUCEENG - 89, Los Angeles, 1989., pp.113-118th.

4) Tomic R.: „Definition of parameters of the structural analysis of generalized orthotropic prismatic beam with aspect distortion functions“, “Technique”, No. 7, XLIX, Belgrade, 1994., pp.36M-40M

5) Tomić R., Maneski T., Nestorovic M. „Optimization of geometric and materia parameters of the orthotropic plate with elliptic openings with respect to stress concentration“, Congress of Mechanics “24.YUMEH”, Belgrade, 2003. (Work on CD)

6) Bosnjak S.:“Neki problemi dinamike i čvrstoće mašina visokih performansi, some of the problems on dynamics and strength of the high performance machines“, Istraživanja i projetovanja za privredu, 1, 2010, Vol 8, broj 27.

7) Gaben Abu Mohamed, Slobodan Krčevinac, Mirko Vujošević: „Modelujući sistemi u optimizaciji”, Istraživanja i projetovanja za privredu, broj 18.

8) Maneski T., Ignjatović D.:“ Dijagnostika čvrstoće konstrukcija u funkciji održavanja”, Istraživanja i projetovanja za privredu, broj 7.

9) Vasić Z., Petrović Z.:“ Savremeni kompozitni materijali u projektovanju i proizvodnji vazduhoplovnih konstrukcija“, Istraživanja i projetovanja za privredu, 2009-Vol.7, broj 4, broj 26.

10) Vasić Z., Petrović Z.:“ Razvoj baze projektnih podataka aviona“, Istraživanja i projetovanja za privredu 3, 2010, Vol8, broj 29.

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