JAES | JOURNAL OF APPLIED ENGINEERING SCIENCE | www.engineeringscience.rs

ISTRAŽIVANJA I PROJEKTOVANJA ZA PRIVREDU

Indexed by

# ON ESTIMATION OF STRESS-STRENGTH RELIABILITY USING LOWER RECORD VALUES FROM ODD GENERALIZED EXPONENTIAL - EXPONENTIAL DISTRIBUTION

***M. O. Mohamed***
*Zagazig University, Faculty of Science, Mathematical Department, Zagazig, Egypt*

***Ahmed H.A.Reda***
*Zagazig University, Faculty of Science, Mathematical Department, Zagazig, Egypt*

***Key words:*** *odd generalize exponential – exponential distribution, lower record, stress-strength model*

**doi:10.5937/jaes0-28710**

***Cite article***:

Mohamed O. M., Reda A. H. A. (2021) ON ESTIMATION OF STRESS-STRENGTH RELIABILITY USING LOWER RECORD VALUES FROM ODD GENERALIZED EXPONENTIAL - EXPONENTIAL DISTRIBUTION, *Journal of Applied Engineering Science*, 19(4), 980 - 988, DOI:10.5937/ jaes0-28710

***Online aceess*** *of full paper is available at: www.engineeringscience.rs/browse-issues*

Original Scientific Paper

doi:10.5937/jaes0-28710

Paper number: 19(2021)4, 878, 980-988

# ON ESTIMATION OF STRESS-STRENGTH RELIABILITY USING LOWER RECORD VALUES FROM ODD GENERALIZED EXPONENTIAL - EXPONENTIAL DISTRIBUTION

***M. O. Mohamed*, Ahmed H.A.Reda***
***Zagazig University, Faculty of Science, Mathematical Department, Zagazig, Egypt***

*This research paper aims to find the estimated values closest to the true values of the reliability function under lower record values, and to know how to obtain these estimated values using point estimation methods or interval estimation methods. This helps researchers later in obtaining values of the reliability function in theory and then applying them to reality which makes it easier for the researcher to access the missing data for long periods such as weather. We evaluated the stress–strength model of reliability based on point and interval estimation for reliability under lower records by using Odd Generalize Exponential–Exponential distribution (OGEE) which has an important role in the lifetime of data. After that, we compared the estimated values of reliability with the real values of it. We analyzed the data obtained by the simulation method and the real data in order to reach certain results. The Numerical results for estimated values of reliability supported with graphical illustrations. The results of both simulated data and real data gave us the same coverage.*

*Key words: odd generalize exponential – exponential distribution, lower record, stress-strength model*

## INTRODUCTION

The lower record values have an important role in solving a lot of problems that concern the studying of missing data for long periods, for example, weather, phenomenon, and health care studies. The statistical study of lower record was introduced. In article [1], to obtain estimators of R(t) and P they don't require Rao-Blackwel Simulation studies and an example based on real data considered as an illustration. for Bayesian comparison of record values based on generalized exponential distribution were considered in [2]. Also, authors found Bayesian analysis for record data Based on Generalized Inverted Exponential Model which considered in [3]. For finding interval estimation for Inverse Rayleigh Distribution based on lower record see [4]. For estimating the reliability for a family of life time distribution based on records see [5]. In [6] they estimated reliability for burr distribution in case of record data. For general class of distribution [7] studied the reliability with lower record. In [8] they found UMVUE of reliability in case of record values and the data has proportional reversed hazard family.

In this article the authors studied the statistical inferences for linedly distribution for record data, see [9]. for other form of linedly distribution was studied by [10] for record data. For bathtub-shaped distribution and record data was studied by [11]. There are more articles about finding reliability as [12] and [13]. This raises some questions: What does the lower record mean? How to study it in different statistical models? And where lower record value can be repressed as xi if its values are less than all previous observations xi<xj for i>j?. To answer these questions, let us see the following definition; Let X1, X2, …. be an infinite sequence of identically and independently distributed (iid) random variables. An observation Xi is called an lower record if Xi<Xj for every i>j. We shall assume that occurs at time i, then the record time sequence is defined as $L_{n\text{-}}$max{i:$X_i<X_{Ln-1}$}. The lower record sequence $R_1,R_2,\dots,R_{nis}$ defined as $R_n=X_{Ln},n\in N$ . The joint probability density function (pdf) of first n lower records is given by:

$$fR_1,R_2,\dots,R_n(r_1,r_2,\dots,r_n)=f(r_n)\prod_{i=0}^{\pi-1}\frac{f(r_i)}{F(r_i)}$$

In a model of Reliability of stress- strength, the service still provides until strength is more than stress. The probability of stress- strength model includes two random variables: X and Y, which indicate strength and stress respectively where R=P(Y<X), and both X and Y are independent. a lot of researches were published in this part of the study of R which explains the major role of probability in statics. for wide application of R=P[Y<X]. A lot of papers studied this model in different situations. The estimation of reliability for parallel system is considered in [13]. In addition, the reliability of the model of stress-strength based on Poisson-exponential distribution which discussed the reliability model based on simulated data [14]. This paper tends to estimate stress-strength model R=(Y<X) where strength and strength are two independent lower record values with OGEE distribution. Assume those scale parameters are known. The importance of OGEE distribution is the flexibility in modeling lifetime data for better representation of the phenomenon contained in the data set. For more information on OGEE distribution see [15]. According to paper [15], the new distribution OGEE can be represented for its Pdf and Cdf

$$f(x)=\lambda\theta e^{\theta x}e^{-\lambda\left(e^{\theta x}-1\right)} \tag{1}$$

*mo11577.mm@gmail.com

$$F(x)=1-e^{-\lambda\left(e^{\theta x}-1\right)}, x>0, \lambda>0, \theta<\infty \tag{2}$$

The maximum likelihood estimate and exact confidence interval of R are derived. Besides, Bayes estimator of R is derived, and all of these estimators are obtained based on mean square errors. The paper is organized as follows. In section (2), maximum and exact C.I, the Bayes estimator and Bootstrap C.I are obtained. For simulation, the studies' proposal is shown in section (3). A real data example is obtained in section (4). Results and discussions are obtained in section (5). tables and figures are represented in section (6). Finally, conclusions appear in section (7).

### Non-Bayesian method

In this section maximum likelihood estimate (MLE) and the exact confidence interval of R is obtained.

### MLE of the Reliability Function R

Let X be the strength of a system or component which is subjected to the stress that X~OGEED ($\theta_1$,β) and Y~OGEED ($\theta_2$,β); therefore, the following reliability function is obtained

$$R=P(Y<X)=\int_0^\infty P[Y<X|Y=y]\,dy$$

$$R=\int_0^\infty 1-e^{-\lambda_1\left(e^{\theta x}-1\right)}\lambda_2\theta e^{x\theta}e^{-\lambda_2\left(e^{\theta x}-1\right)}dx=\frac{\lambda_2}{\lambda_1+\lambda_2} \tag{3}$$

Let r=($r_0$,$r_1$,$r_2$,…,$r_n$) be a set of first observed lower record values of size (n+1) from OGEED with parameter ($\theta_1$,β) and s=($s_0$,$s_1$,$s_2$,…,$s_m$) be an independent set of the observed first lower record values of size (m+1) form OGEED With parameters ($\theta_2$,β), where β assumed to be Learned. The likelihood functions for both observed r and s are given, respectively

$$f(r_n)\prod_{i=0}^{n-1}\frac{f(r_i)}{F(r_i)}=L_1(\theta_1,\beta|\underline{r}) \quad 0<r_n<r_{n-1}<\cdots<r_0<\infty$$

and,

$$g(s_m)\prod_{j=0}^{m-1}\frac{g(s_j)}{G(s_j)}=L_2(\theta_2,\beta|\underline{s}) \quad 0<s_m<s_{m-1}<\cdots s_0<\infty$$

Where F (.) and f (.) are respectively, the cdf and pdf of X and G (.) and g (.) are the cdf and the pdf of y respectively. The likelihood function of the observed record values r and s are obtained,

$$L_1(\lambda_1,\theta|\underline{r})=\lambda_1\theta e^{\theta r_n}e^{-\lambda_1\left(e^{\theta r_n}-1\right)}\prod_{i=0}^{n-1}\frac{\lambda_1\theta e^{\theta r_i}e^{-\lambda_1\left(e^{\theta r_i}-1\right)}}{1-e^{-\lambda_1\left(e^{\theta r_i}-1\right)}} \tag{4}$$

and,

$$L_2(\lambda_2,\theta|\underline{s})=\lambda_2\theta e^{\theta s_m}e^{-\lambda_2\left(e^{\theta s_m}-1\right)}\prod_{j=0}^{m-1}\frac{\lambda_2\theta e^{\theta s_j}e^{-\lambda_2\left(e^{\theta s_j}-1\right)}}{1-e^{-\lambda_2\left(e^{\theta s_j}-1\right)}} \tag{5}$$

The likelihood function of the observed r obtained

$$l_1(\lambda_1,\theta|\underline{r})=Ln(L_1)=(n+1)ln(\lambda_1)+(n+1)ln(\theta)+$$
$$+\theta r_n+(n+1)\lambda_1+\theta\sum_{i=0}^{n}r_i-\lambda_1\sum_{i=0}^{n}r_i-$$
$$-\lambda_1\sum_{i=0}^{n}e^{\theta r_i}-\sum_{i=0}^{n-1}\lambda_1\left(e^{\theta r_i}-1\right)$$

Likewise, the joint log-likelihood function of the observed r and s

$$l=\frac{(\lambda_1\lambda_2\theta)^{n+m+2}e^{\theta(r_n+s_m)}e^{(n+1)\lambda_1+(m+1)\lambda_2}e^{\theta\left(\sum_{i=0}^{n}r_i+\sum_{i=0}^{m}s_i\right)-\lambda_1\sum_{i=0}^{n}e^{\theta r_i}-\lambda_2\sum_{i=0}^{m}e^{\theta s_i}}}{\prod_{i=0}^{n-1}\left(1-e^{-\lambda_1\left(e^{\theta r_i}-1\right)}\right)*\prod_{i=0}^{m-1}\left(1-e^{-\lambda_2\left(e^{\theta s_i}-1\right)}\right)} \tag{6}$$

$$\hat{\lambda}_{ML1}=\frac{(n+1)}{-(n+1)+\sum_{i=0}^{m}e^{\theta s_i}+\sum_{i=0}^{n-1}\left(e^{\theta r_i}-1\right)} \tag{7}$$

Similar for $\lambda_2$

$$\hat{\lambda}_{ML2}=\frac{(m+1)}{-(m+1)+\sum_{i=0}^{m}e^{\theta s_i}+\sum_{i=0}^{m-1}\left(e^{\theta s_i}-1\right)} \tag{8}$$

Thus, the maximum likelihood estimator of $R_{ML}$ is issued as replacements $\lambda_{ML1}$ and $\lambda_{ML2}$ in (3) as follows

$$\hat{R}_{ML}=\frac{\hat{\lambda}_{ML2}}{\hat{\lambda}_{ML1}+\hat{\lambda}_{ML2}} \tag{9}$$

### Confidence Interval of R

The exact confidence interval for R is derived in this subsection as the pdf of $R_n$ is given according to 2:

$$f_{R_n}(r_n)=\frac{1}{F(n+1)}\left[-ln(F(r_n))\right]^n f(r_n) \quad 0<r_n<\infty, n=0,1,2,\ldots$$

$$\frac{1}{F(n+1)}\left[-ln\left(1-e^{-\lambda\left(e^{\theta r_n}-1\right)}\right)\right]^n\lambda\theta e^{-\lambda\left(e^{\theta r_n}-1\right)}$$

$$f_{R_n}(r_n)=\frac{\lambda\theta e^{\theta r_n}e^{-\lambda\left(e^{\theta r_n}-1\right)}(n+1)}{F(n+1)} \tag{10}$$

Therefore, the probability density functions of

$$\hat{\lambda}_1=\frac{n+1}{-2n-1+e^{\sum_{i=0}^{n-1}\theta r_i},e^{\theta r_n}+e^{\theta\sum_{i=0}^{n-1}r_i}}$$

is obtained as follow. Let $Z_1$=

$$\hat{\lambda}_1=\frac{n+1}{-2n-1+e^{\sum_{i=0}^{n-1}\theta r_i},e^{\theta r_n}+e^{\theta\sum_{i=0}^{n-1}r_i}}$$

so the probability density function of $Z_1$ is given as:

$$f(Z_1)=\frac{F(2)e^{\lambda_1(n+1)}}{\lambda_1(n+1)^2}\left[\frac{\lambda_1(n+1)^2g(Z_1)e^{\lambda_1 g(Z_1)(n+1)}}{F(n+1)F(2)}\right]=$$
$$=\frac{F(2)\theta e^{-\lambda_1(n+1)}}{\lambda_1(n+1)^2F(n+1)}\left[\frac{(\lambda_1(n+1))^2}{F(2)}g(Z_1)e^{\lambda_1 g(Z_1)(n+1)}\right], Z_1>0 \tag{11}$$

Where

$$g(Z_1)=\frac{n+1}{Z_1e^{\sum_{i=0}^{n-1}\theta r_i}}+\frac{\left(2n+1-e^{\theta\sum_{i=0}^{n-1}r_i}\right)}{e^{\sum_{i=0}^{n-1}\theta r_i}}$$

$f(Z_1)$ is recognized as the inverted gamma distribution with $[(2,\lambda_1(n+1))]$. Similar, for $Z_2$ has the inverted gamma distribution with $[(2,\lambda_2(m+1))]$.

Therefore Pdf of the reliability (R) can be obtained as follow:

$$\widehat{R}=\frac{1}{1+\frac{Z_1}{Z_2}}$$

Considering, $Z_1/Z_2$, it is easy through the properties of gamma distribution to show that:

$$\frac{2Z_1}{\lambda_1(n+1)}\sim x^2{}_{(2)(2)} \quad \text{and} \quad \frac{2Z_2}{\lambda_2(m+1)}\sim x^2{}_{(2)(2)}$$

Since $Z_1$ and $Z_2$ are independent, then it can be shown that $Z_1/Z_2 \sim \lambda_2/\lambda_1\, F(2((m+1),2(n+1)))$, where $F(2((m+1),2(n+1)))$ is F distribution with $2(m+1),2(n+1)$ degrees of freedom.

Exact distribution of $\hat{R}$ written as:

$$f\left(\widehat{R}\right)=\frac{1}{1+\frac{\lambda_2}{\lambda_1}F\left(2\left((m+1),2(n+1)\right)\right)}$$

A (1-α)% confidence interval for R, based on lower records values is $(L_1, U_1)$, where

$$L_1=\left[1-\frac{Z_1}{Z_2F_{1-\frac{\alpha}{2}}\left(2\left((m+1),2(n+1)\right)\right)}\right]^{-1}$$
$$U_1=\left[1-\frac{Z_1}{Z_2F_{1-\frac{\alpha}{2}}\left(2\left((m+1),2(n+1)\right)\right)}\right]^{-1} \tag{12}$$

Are the lower and upper $\alpha/2^{th}$ percentile point of $F_{2((m+1),2(n+1))}$.

### *Bayes estimate of R Based on MSE*

In this section, the Bayes estimator of R is obtained under mean squared errors.

Firstly, the Bayes estimator for $\lambda_1$ and $\lambda_2$ are obtained, and the non-informative priors for $\lambda_1$ and $\lambda_2$ can be found from the equation of fisher Information as follows:

$$\pi_1(\lambda_1)\propto\frac{1}{\lambda_1} \quad \text{and} \quad \pi_2(\lambda_2)\propto\frac{1}{\lambda_2} \tag{13}$$

The posterior density $\lambda_1$ and $\lambda_2$, denoted by $\pi_1^*\ \lambda_1$ and $\pi_2^*(\lambda_2)$, are obtained by combining the equations of likelihood (3), (4) and the priors of $\lambda_1$ and $\lambda_2$ respectively

$$\pi_1^*(\lambda_1)=\frac{\left[\lambda_1(e^{\theta\sum_{i=0}^{n-1}r_i}-n-1)(2(e^{\theta\sum_{i=0}^{n-1}r_i}-n)-1)\right]^n(e^{-\lambda_1\left(e^{\theta\sum_{i=0}^{n-1}r_i-n-1}\right)}e^{-\lambda_1}\left(2\left(e^{\theta\sum_{i=0}^{n-1}r_i}-n\right)-1\right)}{n!\left(\left(e^{\theta\sum_{i=0}^{n-1}r_i}-n-1\right)^n+\left(2\left(e^{\theta\sum_{i=0}^{n-1}r_i}-n\right)-1\right)^n\right)} \tag{14}$$

$$\pi_2^*(\lambda_2)=\frac{\left[\lambda_2\left(e^{\theta\sum_{j=0}^{m-1}s_j}-m-1\right)\left(2\left(e^{\theta\sum_{j=0}^{m-1}s_j}-m\right)-1\right)\right]^n\left(e^{-\lambda_2\left(e^{\theta\sum_{j=0}^{m-1}s_j}-m-1\right)}+e^{-\lambda_2\left(2\left(e^{\theta\sum_{j=0}^{m-1}s_j}-m\right)-1\right)}\right)}{m!\left(\left(e^{\theta\sum_{j=0}^{m-1}s_j}-m-1\right)^m+\left(2\left(e^{\theta\sum_{j=0}^{m-1}s_j}-m\right)-1\right)^m\right)} \tag{15}$$

The bayes estimators of $\lambda_1$ and $\lambda_2$ under squared errors loss function, denoted by $\hat{\lambda}_{1(SE)}$ and $\hat{\lambda}_{2(SE)}$, are the posterior means which can be obtained as follow:

$$\hat{\lambda}_{1(SE)}=\int_0^\infty \lambda_1\pi_1^*(\lambda_1)d\lambda_1 \quad \text{and} \quad \hat{\lambda}_{2(SE)}=\int_0^\infty \lambda_2\pi_2^*(\lambda_2)d\lambda_2 \tag{16}$$

Therefore the bayes estimator of the R under square error loss function, denoted by $\hat{R}_{(SE)}$ can obtained from Compensation in Eq. (3) as follow:

$$\widehat{R}_{(SE)}=\frac{\hat{\lambda}_{2(SE)}}{\hat{\lambda}_{1(SE)}+\hat{\lambda}_{2(SE)}} \tag{17}$$

### *Bootstrap of Bayes C.I*

To find the Bayes confidence interval, use the method of the bootstrap Confidence interval, see [16](Kotz et al., 2003)[15]. In proposes the bootstrap method as an alternative way to construct a confidence interval. Algorithm of the (1-α)% confidence interval for α by using bootstrap method is illustrated below:

1. 1 Use the estimators value $\hat{\lambda}_{1(SE)}$ and $\hat{\lambda}_{2(SE)}$) to generate N=5000 the bootstrap sample $X_1^*$, $X_2^*$,….,$X_N^*$ and $Y_1^*$,$Y_2^*$,..,$Y_N^*$, then compute the estimated value of $\hat{R}(SE)$ by Bayes which shown in Eq. (17).
2. Calculate the bootstrap MSE by

$$\widehat{MSE}_B=\frac{1}{N}\sum_{i=0}^{N}\left(\bar{R}^{(i)}{}_{(SE)}-\bar{R}_{(SE)}\right)$$

Where N=5000

3. The asymptotic (1-α)% confidence interval is given by

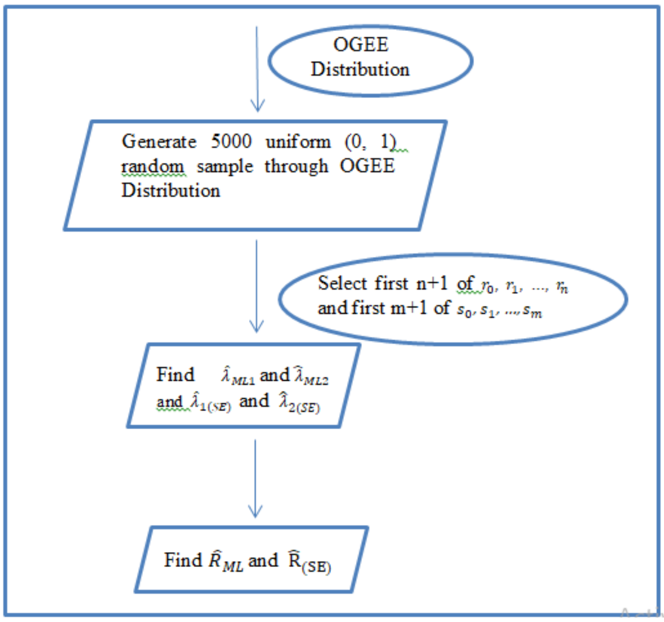
$$\left(\bar{R}_{(SE)}-Z_{\alpha/2}\sqrt{\widehat{MSE}_B},\bar{R}_{(SE)}+Z_{\alpha/2}\sqrt{\widehat{MSE}_B}\right) \tag{18}$$

### *Monte-Carlo Simulation*

In this section a simulation study is designed to find and compare the values of estimated Methods. The exact values of R stress-strength is R=0.531, and R =0.75. The steps of simulation are as follow

1. Generate 5000 uniform (0, 1) random variables and then get the corresponding OGEE random samples of sample size 200 through the transformation technique.
2. Select from each vector the first (n+1), n=2(1)9, lower record values $r_0$, $r_1$, …, $r_n$ for the values of strength random variables X under the assumption that θ is known.
3. Repeat the previous two steps to generate 5000 random samples of size 200 from OGEED and select from each vector the first (m+1), m=2(1)9 lower record values $s_0$, $s_1$, …, $s_m$ for the values of stress random variables Y under the assumption that θ is known.

4. The MLE of $\hat{\lambda}_{ML1}$ and $\hat{\lambda}_{ML2}$ are obtained from (10) & (11), then the MLE of R is obtained by substitute $\lambda_{ML1}$ and $\hat{\lambda}_{ML2}$ in Eq. (12) The exact confidence intervals of R using Eq. (15) are constructed with confidence level at α= 0.05
5. The estimate values of $\hat{\lambda}_{1(SE)}$ and $\hat{\lambda}_{2(SE)}$ by using Bayes estimator under square errors are calculated based on Eq. (19), the Bayes estimates of $\hat{R}_{(SE)}$ under squared error is obtained Eq. (20). Also, the 95% Bootstrap Bayes confidence interval of stress–strength reliability is calculated in equation (21).
6. Compute the average for R, $\hat{R}_{ML}$ and $\hat{R}_{(SE)}$, mean square errors (MSEs), percent of converging, and average probability interval lengths.




*Flowchart of Monte Carlo simulation*

## RESULTS AND DISCUSSION

Simulation results were tabulated in tables (1-6) for estimated values and tables (7,8) for confidence intervals, and represented through figures (1,2).

Tables (1-8)

1. The percentage for MLE is better than the percentage for Bayes.
2. As the size of the sample increase, the MSE becomes lesser.
3. The average length of the bootstrap Bayes confidence interval is smaller than the exact confidence interval, according to tables (7,8).
4. As the sample size increases, the mean confidence interval length becomes lower, except for some points, the average confidence length becomes longer, according to tables (7,8).
5. For some points when the value of m more than n, the percentage increase.

### Tables and figures

### Figures

For simulated data figures [1,2]

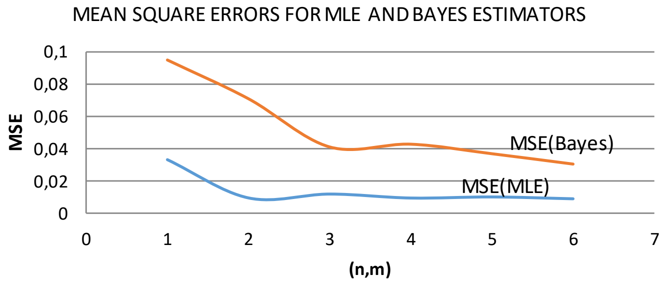



*Figure 1: MSEs OF MLE AND BAYES ESTIMATORS WHEN R=0.531*
*Shows that the MSEs of MLE and BAYES estimators decrease as n and m are increase at R=0.531, also, MSEs of MLE is smallest than MSEs of Bayes*

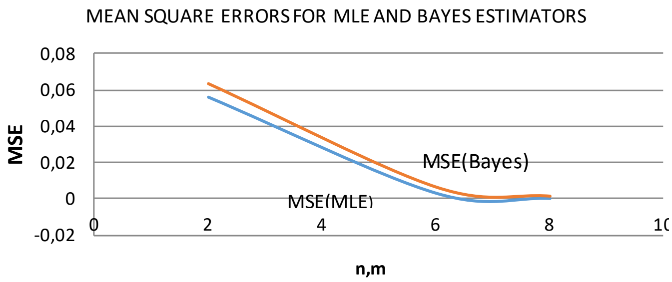



*Figure 2: MSEs OF MLE AND BAYES ESTIMATORS WHEN R=0.75*
*Shows that the MSEs of MLE and BAYES estimators decrease as n and m are increase at R=0.75, also, MSEs of MLE is smallest than MSEs of Bayes*

### Tables

*Table 1: Estimated results for maximum likelihood estimator and Bayes estimators under mean square errors for R at $\lambda_1$=1.7,$\lambda_2$=1.5,$\theta$=0.1*

| n | m | MLE | | | BAYES | | |
|---|---|---|---|---|---|---|---|
| | | $\hat{R}_{ML}$ | MSE | Percent | $\hat{R}_{BMS}$ | MSE | Percent |
| 2 | 1 | 0.491 | 0.03337 | 0.93597 | 0.109 | 0.0948 | 0.205273 |
| | 2 | 0.50 | 0.009774 | 0.93973 | 0.203 | 0.0708 | 0.382298 |
| 3 | 2 | 0.517 | 0.01216 | 0.94162 | 0.443 | 0.041 | 0.834275 |
| | 3 | 0.524 | 0.000525 | 0.98305 | 0.077 | 0.0606 | 0.145009 |
| | 4 | 0.51 | 0.009726 | 0.99623 | 0.108 | 0.0579 | 0.20339 |
| 5 | 4 | 0.521 | 0.009748 | 0.99811 | 0.857 | 0.0206 | 0.99811 |
| | 5 | 0.519 | 0.000498 | 0.99623 | 0.024 | 0.028 | 0.9962 |
| | 6 | 0.522 | 0.009709 | 0.99246 | 0.055 | 0.0427 | 0.992467 |
| 7 | 6 | 0.533 | 0.01038 | 0.98681 | 0.698 | 0.0368 | 0.986817 |
| | 7 | 0.511 | 0.009242 | 0.93051 | 0.12 | 0.0304 | 0.983051 |
| | 8 | 0.510 | 0.009.855 | 0.98116 | 0.26 | 0.0298 | 0.981168 |
| 8 | 7 | 0.544 | 0.0101 | 0.95480 | 0.72 | 0.0295 | 0.954802 |
| | 8 | 0.555 | 0.0097 | 0.99811 | 0.40 | 0.0285 | 0.939736 |
| | 9 | 0.565 | 0.009794 | 0.99623 | 0.615 | 0.0276 | 0.93597 |

*Table 2: Simulation results for maximum likelihood estimator and Bayes estimators under mean square errors for at $\lambda_1$ with θ=0.1*

| n | m | MLE | | | BAYES | | |
|---|---|---|---|---|---|---|---|
| | | $\hat{\lambda}_{ML1}$ | MSE | Percent | $\hat{\lambda}_{BMS1}$ | MSE | Percent |
| 2 | 1 | 0.999 | 0.0237 | 0.587647 | 0.720 | 0.293 | 0.423529 |
| | 2 | 1.339 | 0.04774 | 0.787647 | 0.820 | 0.19 | 0.482353 |
| 3 | 2 | 1.539 | 0.002211 | 0.905294 | 0.920 | 0.19 | 0.541176 |
| | 3 | 1.526 | 0.000225 | 0.897647 | 1.519 | 0.23 | 0.893529 |
| | 4 | 1.626 | 0.001726 | 0.956471 | 1.479 | 0.231 | 0.87 |
| 5 | 4 | 1.622 | 0.000166 | 0.954118 | 1.742 | 0.261 | 0.975294 |
| | 5 | 1.523 | 0.000298 | 0.895882 | 1.229 | 0.243 | 0.722941 |
| | 6 | 1.570 | 0.000550 | 0.923529 | 1.204 | 0.243 | 0.708235 |
| 7 | 6 | 1.666 | 0.003038 | 0.98 | 1.499 | 0.038 | 0.881765 |
| | 7 | 1.666 | 0.000434 | 0.98 | 1.943 | 0.048 | 0.857059 |
| | 8 | 1.678 | 0.000968 | 0.987059 | 1.765 | 0.022 | 0.961765 |
| 8 | 7 | 1.688 | 0.00968 | 0.992941 | 1.778 | 0.089 | 0.954118 |
| | 8 | 1.587 | 0.001275 | 0.933529 | 1.777 | 0.095 | 0.954706 |
| | 9 | 1.612 | 0.00176 | 0.948235 | 1.777 | 0.025 | 0.954706 |

*Table 3: Estimated results for maximum likelihood estimator and Bayes estimators under mean square errors for at $\lambda_2$ with θ=0.1*

| n | m | MLE | | | BAYES | | |
|---|---|---|---|---|---|---|---|
| | | $\hat{\lambda}_{ML2}$ | MSE | Percent | $\hat{\lambda}_{BMS2}$ | MSE | Percent |
| 2 | 1 | 0.714 | 0.971 | 0.476 | 0.556 | 0.383 | 0.370667 |
| | 2 | 0.939 | 1.348 | 0.626 | 0.593 | 0.0873 | 0.395333 |
| 3 | 2 | 0.931 | 1.366 | 0.620667 | 0.987 | 0.0897 | 0.658 |
| | 3 | 1.179 | 1.256 | 0.786 | 0.993 | 0.0758 | 0.662 |
| | 4 | 1.326 | 1.378 | 0.884 | 1.111 | 0.0887 | 0.740667 |
| 5 | 4 | 1.522 | 1.387 | 0.985333 | 1.161 | 0.0891 | 0.774 |
| | 5 | 1.439 | 0.348 | 0.959333 | 1.253 | 0.0873 | 0.835333 |
| | 6 | 1.432 | 0.322 | 0.954667 | 1.276 | 0.089 | 0.788 |
| 7 | 6 | 1.518 | 0.198 | 0.988 | 1.113 | 0.07 | 0.742 |
| | 7 | 1.533 | 0.0397 | 0.978 | 1.249 | 0.089 | 0.832667 |
| | 8 | 1.556 | 0.0402 | 0.962667 | 1.259 | 0.089 | 0.839333 |
| 8 | 7 | 1.515 | 0.0404 | 0.99 | 1.151 | 0.089 | 0.767333 |
| | 8 | 1.533 | 0.0041 | 0.978 | 1.455 | 0.089 | 0.97 |
| | 9 | 1.565 | 0.00412 | 0.956667 | 1.353 | 0.07 | 0.902 |

*Table 4: Estimated results for maximum likelihood estimator and Bayes estimators under mean square errors for R at $\lambda_1$=1, $\lambda_2$=3,θ=1 and R=0.75*

| n | m | MLE | | | BAYES | | |
|---|---|---|---|---|---|---|---|
| | | $\hat{R}_{ML}$ | MSE | Percentage | $\hat{R}_{BMS}$ | MSE | Percentage |
| 2 | 3 | 0.451 | 0.056 | 0.601333 | 0.500 | 0.063 | 0.666667 |
| 3 | 2 | 0.553 | 0.062 | 0.737333 | 0.506 | 0.067 | 0.674667 |
| | 3 | 0.567 | 0.052 | 0.756 | 0.511 | 0.083 | 0.681333 |
| 4 | 3 | 0.642 | 0.002 | 0.856 | 0.513 | 0.091 | 0.684 |
| | 4 | 0.666 | 0.005 | 0.888 | 0.599 | 0.093 | 0.798667 |
| 5 | 4 | 0.640 | 0.006 | 0.853333 | 0.602 | 0.008 | 0.802667 |
| | 5 | 0.576 | 0.003 | 0.768 | 0.650 | 0.006 | 0.866667 |
| 7 | 7 | 0.701 | 0.002 | 0.934667 | 0.665 | 0.005 | 0.886667 |
| | 8 | 0.722 | 0.0001 | 0.962667 | 0.699 | 0.001 | 0.932 |

*Table 5: Estimated results for maximum likelihood estimator and Bayes estimators under mean square errors for at $\lambda_1$ with θ=1*

| n | m | MLE | | | BAYES | | |
|---|---|---|---|---|---|---|---|
| | | $\hat{\lambda}_{ML1}$ | MSE | Percent | $\hat{\lambda}_{BMS1}$ | MSE | Percent |
| 2 | 3 | 0.779 | 0.0830 | 0.779 | 0.539 | 0.0237 | 0.539 |
| 3 | 2 | 0.773 | 0.0860 | 0.773 | 0.526 | 0.0228 | 0.526 |
| | 3 | 0.777 | 0.0860 | 0.777 | 0.626 | 0.0224 | 0.626 |
| 4 | 3 | 0.863 | 0.0898 | 0.863 | 0.627 | 0.0236 | 0.627 |
| | 4 | 0.864 | 0.0808 | 0.864 | 0.777 | 0.0235 | 0.777 |
| 5 | 4 | 0.952 | 0.0871 | 0.952 | 0.716 | 0.0234 | 0.716 |
| | 5 | 0.956 | 0.0801 | 0.956 | 0.806 | 0.0234 | 0.806 |
| 7 | 7 | 0.948 | 0.0907 | 0.948 | 0.913 | 0.0232 | 0.913 |
| | 8 | 0.987 | 0.0021 | 0.987 | 0.953 | 0.0231 | 0.953 |

*Table 6: Estimated results for maximum likelihood estimator and Bayes estimators under mean square errors for at $\lambda_2$ with θ=1*

| n | m | MLE | | | BAYES | | |
|---|---|---|---|---|---|---|---|
| | | $\hat{\lambda}_{ML2}$ | MSE | Percent | $\hat{\lambda}_{BMS2}$ | MSE | Percent |
| 2 | 3 | 2.539 | 0.058 | 0.846333 | 2.406 | 0.028 | 0.802 |
| 3 | 2 | 2.541 | 0.058 | 0.847 | 2.418 | 0.018 | 0.806 |
| | 3 | 2.555 | 0.012 | 0.851667 | 2.610 | 0.021 | 0.87 |
| 4 | 3 | 2.677 | 0.022 | 0.892333 | 2.704 | 0.012 | 0.901333 |
| | 4 | 2.778 | 0.015 | 0.926 | 2.766 | 0.005 | 0.922 |
| 5 | 4 | 2.791 | 0.054 | 0.930333 | 2.777 | 0.004 | 0.925667 |
| | 5 | 2.895 | 0.017 | 0.965 | 2.888 | 0.007 | 0.962667 |
| 7 | 7 | 2.912 | 0.018 | 0.970667 | 2.898 | 0.008 | 0.966 |
| | 8 | 2.951 | 0.019 | 0.983667 | 2.900 | 0.009 | 0.966667 |

*Table 7: Compare between exact confidence intervals and bootstrap bayes confidence interval at $\lambda_1$=1.7,$\lambda_2$=1.5,θ=0.1, R=0.531*

| n | m | Exact CI | | | Bootstrap CI | | |
|---|---|---|---|---|---|---|---|
| | | lower | upper | length | lower | upper | length |
| 2 | 1 | 0.341 | 0.642 | 0.301 | 0.482 | 0.539 | 0.057 |
| | 2 | 0.352 | 0.649 | 0.297 | 0.475 | 0.535 | 0.06 |
| 3 | 2 | 0.146 | 0.411 | 0.265 | 0.509 | 0.539 | 0.03 |
| | 3 | 0.375 | 0.621 | 0.246 | 0.473 | 0.537 | 0.064 |
| | 4 | 0.385 | 0.613 | 0.228 | 0.472 | 0.538 | 0.066 |
| 5 | 4 | 0.237 | 0.473 | 0.236 | 0.493 | 0.535 | 0.042 |
| | 5 | 0.389 | 0.604 | 0.215 | 0.471 | 0.539 | 0.068 |
| | 6 | 0.409 | 0.689 | 0.28 | 0.464 | 0.539 | 0.075 |
| 7 | 6 | 0.42 | 0.629 | 0.209 | 0.468 | 0.533 | 0.065 |
| | 7 | 0.377 | 0.574 | 0.197 | 0.469 | 0.533 | 0.064 |
| | 8 | 0.41 | 0.603 | 0.193 | 0.47 | 0.539 | 0.069 |
| 8 | 7 | 0.42 | 0.608 | 0.188 | 0.469 | 0.533 | 0.064 |
| | 8 | 0.418 | 0.585 | 0.167 | 0.471 | 0.533 | 0.062 |
| | 9 | 0.421 | 0.585 | 0.164 | 0.47 | 0.533 | 0.063 |

*Table 8: Compare between exact confidence intervals and bootstrap bayes confidence interval at $\lambda_1$=1, $\lambda_2$=3,$\theta$=1,R=0.75*

| n | m | Exact CI | | | Bootstrap CI | | |
|---|---|---|---|---|---|---|---|
| | | lower | upper | length | lower | upper | length |
| 2 | 1 | 0.541 | 1.746 | 1.205 | 0.44 | 0.76 | 0.32 |
| | 2 | 0.542 | 1.846 | 1.304 | 0.47 | 0.77 | 0.3 |
| 3 | 2 | 0.544 | 1.555 | 1.011 | 0.454 | 0.788 | 0.334 |
| | 3 | 0.662 | 1.725 | 1.063 | 0.476 | 0.799 | 0.323 |
| | 4 | 0.675 | 1.653 | 0.978 | 0.47 | 0.753 | 0.283 |
| 5 | 4 | 0.698 | 1.887 | 1.189 | 0.454 | 0.801 | 0.347 |
| | 5 | 0.72 | 1.788 | 1.068 | 0.472 | 0.833 | 0.361 |
| | 6 | 0.747 | 1.746 | 0.999 | 0.44 | 0.840 | 0.4 |
| 7 | 6 | 0.688 | 1.877 | 1.189 | 0.323 | 0.833 | 0.51 |
| | 7 | 0.658 | 1.777 | 1.119 | 0. 331 | 0.85 | 0.519 |
| | 8 | 0.677 | 1.677 | 1.00 | 0.321 | 0.84 | 0.519 |
| 8 | 7 | 0.699 | 1.554 | 0.855 | 0.322 | 0.876 | 0.554 |
| | 8 | 0.608 | 1.407 | 0.799 | 0.337 | 0.850 | 0.513 |
| | 9 | 0.619 | 1.391 | 0.772 | 0.326 | 0.865 | 0.539 |

### *CONCLUSION*

In this article, the MLE and Bayes estimators were computed to stress - strength reliability function when both the stress and the strength have GEE distributions based on lower record values. Furthermore, the exact confidence interval and bootstrap Bayes confidence interval of R were derived.

Overview of the estimated results obtained in the previous tables in which we find that the percentage of convergence for MLE is better than the percentage of affinity for Bayes. MSEs for MLE are less than MSEs of Bayes. The average confidence interval lengths for Bootstrap Bayes are shorter than the average confidence interval lengths corresponding to the MLE.

Regarding the number of record values n and m, it is observed that MSEs and average lengths of time decrease when the number of record values n and m increases.

Based on the aforementioned, MLE is better than Bayes estimations within the square error loss function in terms of percent of converging. Although confidence intervals have been examined, Bootstrap Bayes are better estimated as the lengths of time intervals are shorter.

### *FUNDING STATEMENT*

The author(s) received no specific funding for this study.

### *CONFLICTS OF INTEREST*

The authors declare that they have no conflicts of interest to report regarding the present study.

### *REFERENCE*

1. A. Chaturvedi and A. Malhotra, “On Estimation of Stress-Strength Reliability Using Lower Record Values from Proportional Reversed Hazard Family,” American Journal of Mathematical and Management Sciences, vol. 39, no. 3, pp. 234–251, Jul. 2020, doi: 10.1080/01966324.2020.1722299.
2. A. Hassan, H. Muhammed, and M. Saad, “Estimation of Stress-Strength Reliability for Exponentiated Inverted Weibull Distribution Based on Lower Record Values,” BJMCS, vol. 11, no. 2, pp. 1–14, Jan. 2015, doi: 10.9734/BJMCS/2015/19829.
3. A. S. Hassan, M. Abd-Allah, and H. F. Nagy, “Bayesian Analysis of Record Statistics Based on Generalized Inverted Exponential Model,” International Journal on Advanced Science, Engineering and Information Technology, vol. 8, no. 2, Art. no. 2, Mar. 2018, doi: 10.18517/ijaseit.8.2.3506.
4. B. Tarvirdizade and H. Kazemzadeh Garehchobogh, “Interval Estimation of Stress-Strength Reliability Based on Lower Record Values from Inverse Rayleigh Distribution,” Journal of Quality and Reliability Engineering, vol. 2014, pp. 1–8, 2014, doi: 10.1155/2014/192072.
5. A. Chaturvedi and A. Malhotra, “Estimation and testing procedures for the reliability functions of a family of lifetime distributions based on records,” Int J Syst Assur Eng Manag, vol. 8, no. S2, pp. 836–848, Nov. 2017, doi: 10.1007/s13198-016-0531-2.
6. A. Chaturvedi and S. Vyas, “Estimation and Testing Procedures for the Reliability Functions of Three Parameter Burr Distribution under Censorings,” Statistica, vol. Vol 77, pp. 207-235 Pages, Jan. 2018, doi: 10.6092/ISSN.1973-2201/6965.

7. F. Condino, F. Domma, and G. Latorre, “Likelihood and Bayesian estimation of $$P(Y{<}X)$$ P (Y<X) using lower record values from a proportional reversed hazard family,” Stat Papers, vol. 59, no. 2, pp. 467–485, Jun. 2018, doi: 10.1007/s00362-016-0772-9.
8. M. J. S. Khan and Mohd. Arshad, “UMVU Estimation of Reliability Function and Stress–Strength Reliability from Proportional Reversed Hazard Family Based on Lower Records,” American Journal of Mathematical and Management Sciences, vol. 35, no. 2, pp. 171–181, Apr. 2016, doi: 10.1080/01966324.2015.1134363.
9. A. Asgharzadeh, A. Fallah, M. Z. Raqab, and R. Valiollahi, “Statistical inference based on Lindley record data,” Stat Papers, vol. 59, no. 2, pp. 759–779, Jun. 2018, doi: 10.1007/s00362-016-0788-1.
10. A. Pak and S. Dey, “Statistical Inference for the power Lindley model based on record values and inter-record times,” Journal of Computational and Applied Mathematics, vol. 347, pp. 156–172, Feb. 2019, doi: 10.1016/j.cam.2018.08.012.
11. M. Z. Raqab, O. M. Bdair, and F. M. Al-Aboud, “Inference for the two-parameter bathtub-shaped distribution based on record data,” Metrika, vol. 81, no. 3, pp. 229–253, Apr. 2018, doi: 10.1007/s00184-017-0641-0.
12. A. Sadeghpour, M. Salehi, and A. Nezakati, “Estimation of the stress–strength reliability using lower record ranked set sampling scheme under the generalized exponential distribution,” Journal of Statistical Computation and Simulation, vol. 90, no. 1, pp. 51–74, Jan. 2020, doi: 10.1080/00949655.2019.1672694.
13. A. Iranmanesh, K. Fathi Vajargah, and M. Hasanzadeh, “On the estimation of stress strength reliability parameter of inverted gamma distribution,” Math Sci, vol. 12, no. 1, pp. 71–77, Mar. 2018, doi: 10.1007/s40096-018-0246-4.
14. C. Zhang and Y. Zhang, “Common cause and load-sharing failures-based reliability analysis for parallel systems,” EiN, vol. 22, no. 1, pp. 26–34, Dec. 2019, doi: 10.17531/ein.2020.1.4.
15. M. O. Mohamed, “Reliability with Stress-Strength for Poisson-Exponential Distribution,” j comput theor nanosci, vol. 12, no. 11, pp. 4915–4919, Nov. 2015, doi: 10.1166/jctn.2015.4459.
16. S. Vv, “New Odd Generalized Exponential - Exponential Distribution: Its Properties and Application,” BBOAJ, vol. 6, no. 3, Apr. 2018, doi: 10.19080/BBOAJ.2018.06.555686.
17. S. Kotz, Y. Lumelskii, and M. Pensky, The Stress-Strength Model and its Generalizations - Theory and Applications. World Scientific Publishing Co. Pte. Ltd., 2003.
18. “Smith, R.L. and Naylor, J. (1987) A Comparison of Maximum Likelihood and Bayesian Estimators for the Three-Parameter Weibull Distribution. Applied Statistics, 36, 358-369. - References - Scientific Research Publishing.” https://www.scirp.org/(S(vtj3fa45qm1ean45vvffcz55))/reference/ReferencesPapers.aspx?ReferenceID=1287256 (accessed Sep. 16, 2020).

*Paper submitted: 05.10.2020.*
*Paper accepted: 07.04.2021.*