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ANALYSIS AND EVALUATION OF FLOOD ROUTING USING MUSKINGUM METHOD

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There are several mathematical procedures that deal with hydrologic flood routing. The Muskingum technique is one of the most common techniques used for flood routing for river reach. From the hydrologic point of view, flood routing in a stream is used to predict the flood discharge, or storage, at any downstream station in a stream channel from a known discharge, or stage, at an upstream station. Hydrologic routing is an approximate technique. However, it provides relatively easy alternative, for solving flood routing problems. It is based on the storage and the continuity equations. In principle hydrologic routing employs historical data on inflow and outflow discharges in the reach under study. The Muskingum method is the particular one to be considered in this paper, describing three procedures, other than the classical trial and error procedure, for solving flood routing.

Keywords: Muskingum method, hydraulic routing, hydrologic routing, K, x

1 INTRODUCTION

Flood routing is defined as “a technique used to compute the system storage and system dynamics on the shape and movement of the flood wave” [1-3]. Flood routing, applied to reservoirs or to stream flow, can be classified into the following two main categories according to the basis of routing:

1. Hydraulic Routing: It is the most accurate technique in flood routing since it is based on the equations of motion. Therefore, this method relies on the solution of the differential equations of motion for unsteady flow and the continuity equation. The method of characteristics and the diffusion analogy method are two examples of this kind of routing [4-8].
2. Hydrologic Routing: It is an approximate technique but provides relatively alternative easy for solving flood routing problems. It is based on the storage and the continuity equations. In principle hydrologic routing employs historical data on inflow and outflow discharges in the reach under study. These data are more easily available than those required for the hydraulic routing. Muskingum method is a particular for the hydrologic routing [9-12].

A consideration of all possible methods of hydraulic routing is beyond the scope of this paper. The Muskingum method is the particular one to be considered in this study, describing three procedures, other than the classical trial and error procedure, for solving the problem. From the hydrologic point of view, flood routing in a stream is used to predict the flood discharge, or storage, at any downstream station in a stream channel from a known discharge, or stage, at an upstream station [13-15]. The Muskingum method was developed by McCarthy [16] in connection with the planning of flood control reservoirs in the Muskingum River basin, Ohio, USA, by the corps of engineering [17, 18]. Firstly, it is a simple method, which can be used for flood routing without much complication as far as the procedural details are concerned. Secondly, its parameters can be calculated using the record of past historical floods. It does not require a knowledge of the riverbed geometry as the phenomenon can be reproduced well enough based on the calibration carried out using experimental data relative to the extreme sections of most significantly long reaches.

2 TOOLS USED IN MUSKINGUM METHOD

There are three main tools used in the Muskingum method. These are:

1. Inflow and outflow hydrographs.
2. Continuity equation; and,
3. The storage equation.

2.1 Inflow and Outflow Hydrographs

A hydrograph is “a graph of discharge, or, stage passing a particular point on a stream, plotted as a function of time.” In flood routing by the Muskingum method, an inflow and an outflow hydrograph are needed [19 - 21]. The shape of the outflow hydrograph differs from the inflow one in two ways:

1. The outflow hydrograph has a lower peak than that of the inflow, since the volume of water interring the reach as inflow, is dissipated as infiltrated and evaporated water; and,

2. The peak of the outflow hydrograph occurs later since the flood wave needs some time to travel between the upstream end (where inflow is measured) and the downstream (where outflow is measured) of the reach. This delay is important in flood control.

2.2 Continuity Equation

Considering the reach shown in Figure 1 the following equation represents the unsteady flow in the reach.

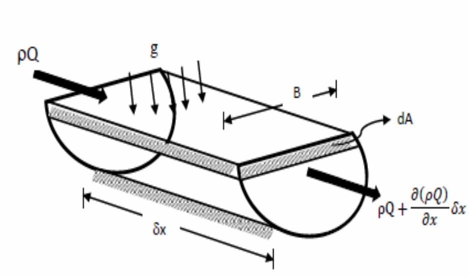


Figure 1. Flow through a reach

net inward flux = change in volume

or:

$$\rho Q + \rho\, g(P+B)\delta X - \left(\rho Q + \frac{\partial(\rho Q)}{\partial x}\partial x\right) = \rho \frac{\partial A}{\partial t}\,\partial X \quad (1)$$

Where:

ρ = fluid density

Q = discharge into the reach

g = gain per unit area of lateral surface

P = wetted parameter of the cross section

B = top width of the cross section

δx = length of the reach

A = area of cross section.

Eq. (1) may be written after simplification as:

$$\frac{\partial(\rho Q)}{\partial x} + \rho\frac{\partial A}{\partial t} = \rho\, g(P+B) = \rho G \quad (2)$$

where:

G = gain per unit length of the reach.

2.3 Storage Equation

Assuming that the flow is changing gradually with time, and that the water surface profile of the reach, as shown in Figure 2 is a straight line, the prism storage can be represented as a function of outflow in the form

Prism Storage = KO

where: K is constant with the dimension of time.

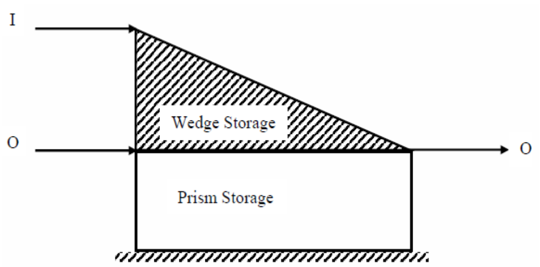


Figure 2. Prism and wedge storage in a reach.

Also, the wedge storage can be expressed as a function of the instantaneous values of the inflow and the outflow as:

Wedge Storage = *C (I – O) = K x (I – O)* (3)

where:

x is a positive fraction less than 1.0.

Taking the total storage by summing up the prism storage and the wedge storage. Or:

$$S = K\,O + K\,x\,(I - O) \quad (4)$$

or

$$S = K\,[\,x\,I + (1 - x)\,O] \quad (5)$$

NOTICE: that the actual. storage equation is not a direct function with I and O but it is a direct function with I^n and O^n since the water surface profile will not be a straight line. Therefore, the actual storage equation will be in the form:

$$S = K\,[x\,I^n + (1\text{-}x)\,O^n] \quad (6)$$

However, for simplicity n is taken as unity.

Where:

K = storage time constant for the reach; it is equal to time required for a flood wave to be traveled through the reach approximately.

x = is a dimensionless constant known as the weighing factor.

Both K and x are constant for a certain reach since they depend mainly on the hydraulic characteristics of the channel section. But both K and x are considered to vary in time and space according to the flow variability. Values of K and x are constants all over this study.

3 STORAGE CALUCATION FOR THE REACH

To proceed in Muskingum Method, we must find values of reach storage at each given interval. From the given data of inflow and outflow discharges and the use of the continuity equation we can find approximate values for the storage volumes in the reach as:

$$I_{average} - O_{average} = \frac{\partial S}{\partial t} \quad (7)$$

Which is the same as:

$$(I_t + I_{t+\partial t}) - (Q_t + Q_{t\partial t}) = \frac{S_{t+\partial t} - S_t}{0.5\,S_t} \quad (8)$$

So, we can write for the storage at the end of the time interval δt:

$$S_{t+\partial t} = S_t + (0.5\,S_t)[(I_t + I_{t+\partial t}) - (Q_t + Q_{t+\partial t})] \quad (9)$$

Storage at the end of each time interval as easily calculated if we know the initial storage volume for the reach (So). Note that we can write the following expression for the storage at any time when we have the data at equal time interval (∂t)

$$S_n = S_O + (\partial t)\,[0.5\,(I_O + I_n - O_O - O_n) + \sum_{i=1}^{n-1}(I_i - O_i)] \quad (10)$$

But for simplicity, sometimes we can use the following expression for the storage by assuming that:

$$0.5(I_o + I_n - O_o - O_n) \quad (11)$$

can be neglected with respect to $\sum_{i=1}^{n-1}(I_i - O_i)$

$$S_n = S_O + (\partial t)\,[\sum_{i=1}^{n-1}(I_i - O_i)] \quad (12)$$

Therefore, the first expression is used since it is more accurate.

4 SOLUTION FOR THE PARAMETERS K AND X

In the storage equation, Eq. (5), both I and O are given as the inflow and outflow hydrographs respectively. S can be calculated from the previous section. The remaining unknowns are the storage time constant K and the weighing factor x.

The most known procedure to find values of K and x is the procedure developed by McCarthy et al [16]. It may be summarized as follows:

- Considering the storage Eq. (5) and defining a new item which is known as the weighted discharge factor, W_f where:

$$W_f = x\,I + (1 - x)\,O \quad (13)$$

So the storage equation may be written as:

$$S = K\,W_f \tag{14}$$

Since K is equal to the time required for a flood wave to travel from the upstream of the reach to the downstream end of it; an approximate value of K may be expressed as the time difference in occurring the inflow peak and the outflow one.

5 OTHER METHODS FOR SOLVING K AND X

Three other methods will be developed in this study for finding the values of K and x other than the trial-and-error procedure mentioned previously (section 4).

5.1 Method one

Considering the storage Eq. (5), then taking the derivative of this equation with respect to time (t), will yield:

$$\frac{\partial S}{\partial t} = K\left[x\,\frac{\partial I}{\partial t} + (1-x)\,\frac{\partial O}{\partial t}\right] \tag{15}$$

Since the continuity equation is:

$$\frac{\partial S}{\partial t} = I - O \tag{16}$$

At point t_B where the inflow hydrograph intersects the outflow hydrograph (Figure 3), I = O

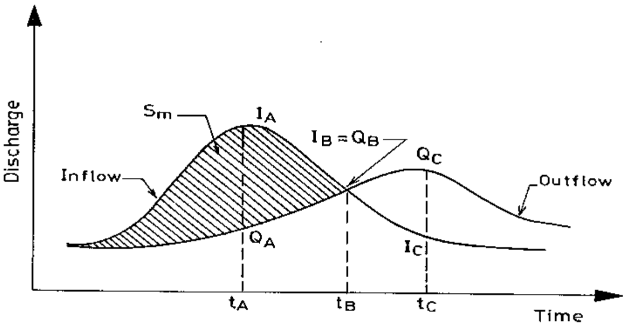


Figure 3. Inflow and outflow hydrographs for a river reach.

Substituting this value (I = O) into Eq. (5), would give:

$$\left(\frac{\partial S}{\partial t}\right)_{at\ point\ A} = 0 \tag{18}$$

Therefore, Eq. (15) can be written for point A as:

$$x\,\frac{\partial I}{\partial t} + (1-x)\frac{\partial O}{\partial t} = 0 \tag{19}$$

or

$$x = \frac{\partial O/\partial t}{\left(\partial O/\partial t\right) - \left(\partial I/\partial t\right)} \tag{20}$$

where:

$\partial I/\partial t$: the slope of the tangent to the inflow hydrograph at the point of intersection (point A).

$\partial O/\partial t$: is the slope of the tangent to the outflow hydrograph at the point of intersection.

Considering the storage equation again, Eq. (5), at the point of intersection between the inflow and the outflow hydrographs,

I = 0 and S = Maximum Storage

Substituting I for 0 and S_m for S in the storage equation.

$$S_m = K\,[xI + (1 - x)\,I] \tag{21}$$

Simplifying Eq. (10) would yield:

$$S_m = KI \tag{22}$$

Or if written in terms of K

$$K = S_m/I \tag{23}$$

5.1.1 Procedure of solution to method one

To find the values of the parameter, K and x using this method the following steps must followed:

- Plot the inflow and outflow hydrographs.
- At the intersection point of the two hydrographs, draw a tangent line for the inflow and outflow hydrograph.
- Find the slopes of both tangents, $({}^{\partial I}/_{\partial t}\ and\ {}^{\partial O}/_{\partial t})$
- Calculate the value of x using Eq. (20)
- Find the maximum storage value, S_m, by measuring the area bounded by the inflow and the outflow hydrographs from time equals zero to the time when both inflows at outflow hydrographs intersects.
- Find the value of the discharge at the point of intersection.
- Calculate the value of K using Eq. (23).

In the actual storage equation, Eq. (6), n is assumed to be unity in Muskingum Method. However, it differs from unity. This method makes easy to find the exact value of n**.** The procedure of finding n, needs two inflow and outflow hydrographs for the same reach taken for different floods. The procedure to find the value of n (since at maximum storage, I = O) is:

$$S_m = K\,I^n \tag{24}$$

For each inflow and outflow hydrographs a value of I is founded while I = 0. Two equations may be written as follows:

$$S_{m1} = K\,I_1^n \tag{25}$$

$$S_{m2} = K\,I_2^n \tag{26}$$

where:

K and n are constants for the same reach.

S_{m1} and S_{m2} are the maximum storage for the first and the second flood respectively.

I_1 and I_2 are the inflows when I = 0 for the first and the second flood respectively.

If Eq. (24) is to be written in the logarithm form it will be as:

$$Log\,(S_m) = Log\,(K\,I^n) = log\,(K) + n\,log(I) \tag{27}$$

Therefore, Eqs. (25) and (26) can be written as:

$$Log\,(S_{m1}) = Log(K) + n\,log(I_1),\ \text{and} \tag{28}$$

$$Log\,(S_{m2}) = log(K) + n\,Log(I_2) \tag{29}$$

If Eq. (29) is subtracted from Eq. (28) the result will be:

$$Log\,(S_{m1}) - Log\,(S_{m2}) = n\,[Log(I_1) - Log(I_2)] \tag{30}$$

Which can be written in the form:

$$n = \frac{\log\left({}^{S_{m1}}/_{S_{m2}}\right)}{\log\left({}^{I_1}/_{I_2}\right)} \tag{31}$$

5.2 Method Two

Considering the storage Eq. (5) and considering the inflow and outflow hydrographs as shown in Figure 6. Taking the derivative of the storage with respect to time

$$\frac{\partial S}{\partial t} = K\left[x\,\frac{\partial I}{\partial t} + (1-x)\,\frac{\partial O}{\partial t}\right] \tag{32}$$

However, with reference to the continuity equation, then we could write Eq. (13) as:

$$(I - O) = K\left[x\,\frac{\partial I}{\partial t} + (1-x)\,\frac{\partial O}{\partial t}\right] \tag{33}$$

Applying Eq. (33) to the point of $t = t_B$; where

$$\frac{\partial I}{\partial t} = 0$$

Therefore, Eq. (33) will be:

$$K - Kx = \frac{I_B - O_B}{\left(\frac{\partial O}{\partial t}\right)_B} \tag{34}$$

If we let

$$\frac{I_B - O_B}{\left(\frac{\partial O}{\partial t}\right)_B} = B \tag{35}$$

Then Eq. (33) can be written as:

$$K - Kx = B \tag{36}$$

Applying Eq. (34) to the point of t = t_c; where

$$\frac{\partial O}{\partial t} = 0$$

Eq. (34) will be

$$Kx = \frac{I_C - O_C}{\left(\frac{\partial I}{\partial t}\right)_C} \tag{37}$$

Let

$$\frac{I_C - O_C}{\left(\frac{\partial I}{\partial t}\right)_C} = C \tag{38}$$

Then Eq. (38) can be written as:

$$K\,x = C \tag{39}$$

Solving Eqs. (36) and (39) simultaneously:

$$K = B + C \tag{40}$$

$$x = \frac{C}{B+C} \tag{41}$$

where B and C are defined by Eqs. (35) and (38) respectively.

5.2.1 Solution Procedure

The parameters K and x can be found as follows:

- For the plotted inflow and outflow hydrographs, define points B and C which are the peaks of the inflow at outflow hydrographs respectively.
- For point B determine the values of the inflow and outflow discharges, IB and OB, and the slope of the outflow hydrographs, $\left(\frac{\partial O}{\partial t}\right)_B$
- Calculate the value of B using Eq. (35)
- For point C determine the values of the inflow and outflow discharges, I_c and O_c, and the slope of the inflow hydrographs, $\left(\frac{\partial I}{\partial t}\right)_C$
- Calculate the value of C using Eq. (38)
- Calculate the value of K using Eq. (46)
- Calculate the value of x using Eq. (41).

5.2.2 Application to Muskingum Method

The continuity equation, can be written in an average form as:

$$\frac{S_{t+\partial t} - S_t}{\partial t} = \left(\frac{I_t + I_{t+\partial t}}{2}\right) - \left(\frac{O_t + O_{t+\partial t}}{2}\right) \quad , \tag{42}$$

and the storage Eq. (5) it can be written in the form of:

$$S_{t+\partial t} - S_t = K\left[x\left(I_{t+\partial t} - I_t\right) + (1-x)(O_{t+\partial t} - O_t)\right] \tag{43}$$

Combining Eqs. (42) and (43) would yield:

$$0.5\,\partial t\,[(I_{t+\partial t} + I_t) - (O_{t+\partial t})] = K\,[x\,(I_{t+\partial t} - I_t) + (1-x)(O_{t+\partial t} - O_t)] \quad (44)$$

Simplifying the above Eq. (44)

$$[(1-x)\,K + 0.5\partial t]O_{t+\partial t} = [Kx + 0.5\partial t]I_t + [-Kx + 0.5\partial t]I_{t+\partial t} + [(1-x)\,K + 0.5\partial t]O_t \quad (45)$$

I.e.

$$O_{t+\partial t} = \left[\frac{Kx+0.5\partial t}{-Kx+0.5\partial t}\right] I_t + \left[\frac{0.5\partial t-Kx}{0.5\partial t+K-Kx}\right] I_{t+\partial t} + \left[\frac{-0.5\partial t+K-Kx}{0.5\partial t+K-Kx}\right] O_t \quad (46)$$

$$O_{t+\partial t} = C_1\,I_t + C_2\,I_{t+\partial t} + C_3\,O_t \quad (47)$$

where C_1, C_2 and C_3 are constants depending on values of K, x and the time interval, δt.

$$C_1 = \frac{0.5\partial t+Kx}{0.5\partial t+K-Kx} \quad (48)$$

$$C_2 = \frac{0.5\partial t+Kx}{0.5\partial t+K-Kx} \quad (49)$$

$$C_3 = \frac{0.5\partial t+Kx}{0.5\partial t+K-Kx} \quad (50)$$

Therefore, the main advantages of the two methods explained previously are:

- They deal with the inflow and the outflow hydrographs directly.
- They do not use the trial-and-error procedures.
- They give a more accurate value of x.

5.2.3 Notes on Method One and Two

Both Method One and Method Two, give very crude solutions for K and x if the flood is composed of more than one flood wave. That is, if for a flood wave begins before the first flood demolishes, so the assumption of natural decay for the hydrographs will not be valid and complex hydrographs will result.

In the case of method two, the accuracy of the solution of using these methods in the case of complex hydrographs will depend on how late the second flood will occur. This is because method two depends on more extended field data than method one, i.e. Method Two needs data that do not belong to the first flood wave, but they are affected by both the first and the second flood.

5.3 Method three

In this method, the storage equation Eq. (5) is divided by the outflow, O, where the result will be:

$$\frac{S}{O} = K\,x\,\left(\frac{I}{O}\right) + K\,(I - x) \quad (51)$$

Since both K and x are constants, Eq. (51) is an equation of a straight line. If a graph of (S/0) is plotted against (I/O) the result will be a straight line with a slope of:

$$m = K\,x \quad (52)$$

intersection with the vertical, (S/0), b will be:

$$b = K\,(I - x) \quad (53)$$

Solving for K and x:

$$X = \frac{m}{b+m} \quad (54)$$

$$K = b + m \quad (55)$$

However, in this method, the resulting relationship of (S/O) versus (I/O), is not, in any way, a straight line. Therefore, an approximation for a straight line will be considered through some point.

6 PRACTICAL CASE STUDIES

A sample data set of Hammer and Mckichan [22] is used to illustrate the calculations made for method one and two. A second data set given by Hjelmfelt and Cassidy [23] is used for the calculations for method three.

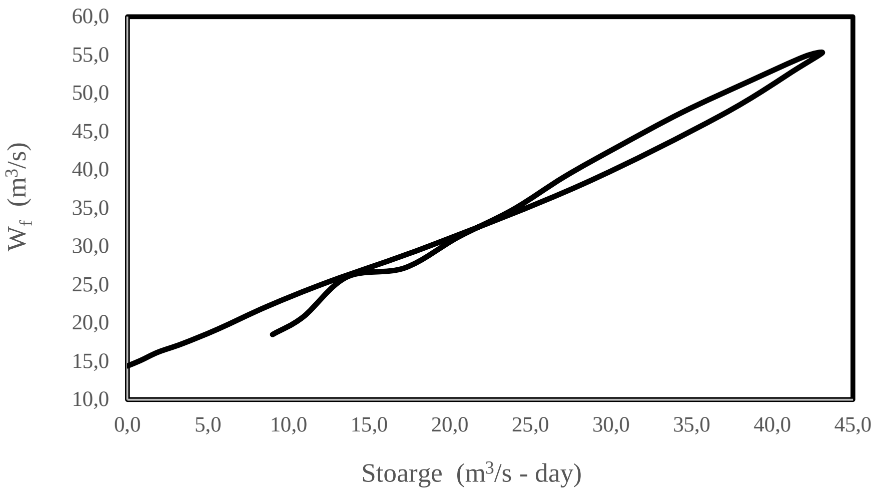
6.1 Method One and Two

The data used for method one and two are given in Table 1.

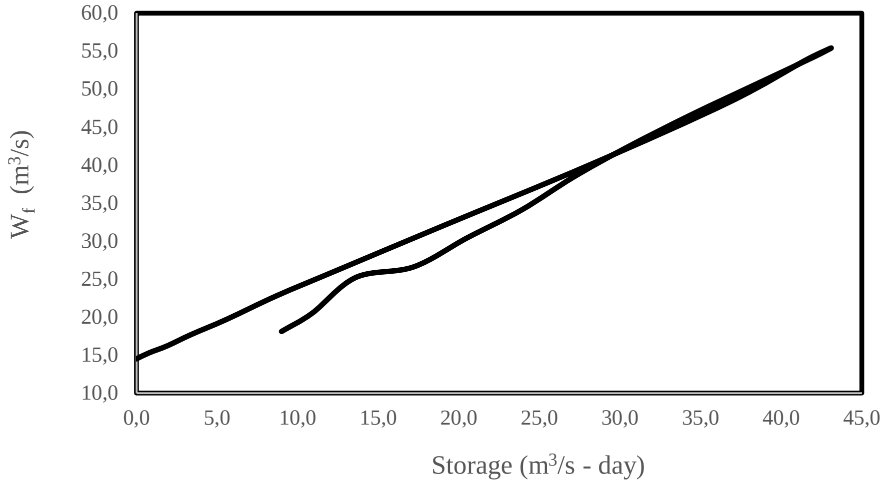
Table1. Actual inflow and outflow data for method one and two*

Time (days)	I (m^3/s)	O (m^3/s)	S (m^3/s – day)	W_f (m^3/s) x = 0.1	W_f (m^3/s) x = 0.2	W_f (m^3/s) x = 0.3
0.0	15.5	14.1	0.0	14.4	14.5	14.7
0.5	16.9	14.9	0.8	15.1	15.3	15.5
1.0	18.4	15.9	1.9	16.2	16.2	16.7
1.5	20.4	17.9	3.4	17.3	17.7	18.0
2.0	24.1	18.6	5.6	19.2	19.7	20.3
2.5	28.2	21.5	8.7	22.2	22.8	23.5
3.0	33.1	24.3	12.5	25.4	26.2	27.1
3.5	45.3	28.5	18.9	30.2	31.9	33.5
4.0	56.6	36.0	28.2	38.1	40.1	42.2
4.5	59.5	45.7	36.8	47.1	48.3	49.8
5.0	58.1	52.8	41.6	53.3	53.9	54.4
5.5	56.1	55.2	43.1	55.3	55.4	55.5
6.0	49.8	55.3	42.0	54.8	54.2	53.7
6.5	44.5	52.4	38.6	51.6	50.8	50.0
7.0	39.4	48.4	34.4	47.5	46.6	45.7
7.5	36.8	43.7	30.4	43.0	42.3	41.6
8.0	33.4	39.7	27.1	39.1	38.4	37.8
8.5	28.9	35.4	23.9	34.8	34.1	33.5
9.0	24.6	31.9	20.5	31.2	30.4	29.7
9.5	21.8	27.8	17.2	27.2	26.6	26.0
10	18.7	25.0	13.6	26.0	25.2	24.4
10.5	16	21.1	10.9	20.8	20.5	20.2
11	14.4	19.0	9.0	18.5	18.1	17.6

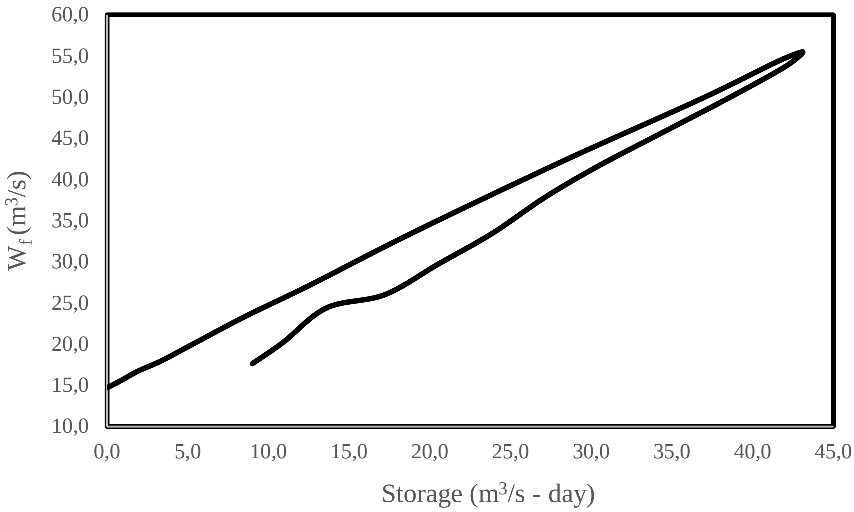
* Hammer and Mckichan [22]



(a)



(b)



(c)

Figure 4. Storage-weighted flow relationship: (a) x = 0.1, (b) x = 0.2, (c) x = 0.3.

The plots of storage versus weighted flow for x, 0.1, 0.2 and 0.3 are shown in Figure 4, respectively. In examining this figure (Figure 4a, b, c), it is clear that x = 0.2 gives the narrowest loop in the storage versus weighted flow graph and K = 1 day. To find the values of x and K using method one, we use the hydrograph in Figure 5 and the previous equations given in section 5. The values of x and K are 0.16 and 0.8 day respectively. However, for method two, the x value 0.19 and K = 1 day.

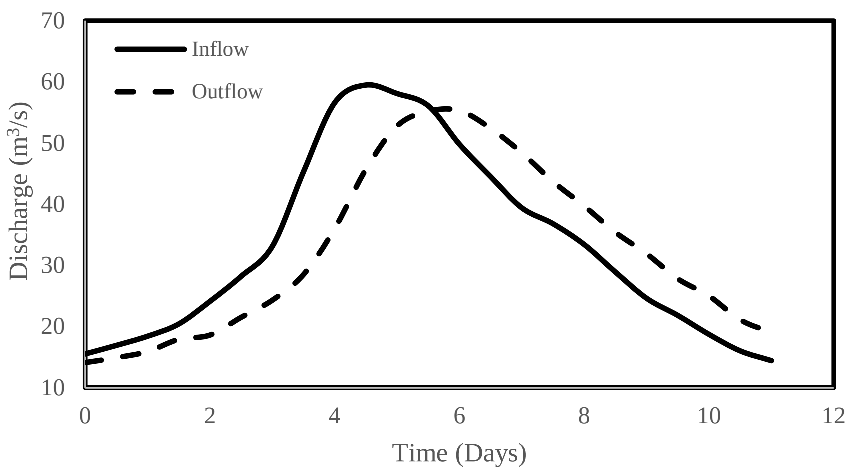


Figure 5. Inflow and outflow at the reach.

6.2 Method three

A sample data set of Hjelmfelt and Cassidy [23] is used to illustrate the calculations made for method three. The data is shown in Table 2.

Table2. Actual inflow and outflow data for method three*

Time (days)	I (1000 cfs)	O (1000 cfs)	S (1000 cfs)	I/O	S/O
0.0	2.2	2.0	0.0		
0.5	14.5	7.0	1.9	1.86	0.43
1.0	28.4	11.7	8.0	2.29	0.85
1.5	31.8	16.5	16.0	2.14	1.13
2.0	29.7	24.0	21.2	1.52	1.05
2.5	25.3	29.1	21.7	1.04	0.82
3.0	20.4	28.4	18.8	0.80	0.65
3.5	16.3	23.8	14.9	0.70	0.57
4.0	12.6	19.4	11.3	0.67	0.52
4.5	9.3	15.3	8.1	0.63	0.47
5.0	6.7	11.2	5.5	0.60	0.41
5.5	5.0	8.2	3.6	0.60	0.37
6.0	4.1	6.4	2.2	0.62	0.30
6.5	3.6	5.2	1.2	0.66	0.21
7.0	2.4	4.6	0.2	0.61	0.05

*Hjelmfelt and Cassidy [23]

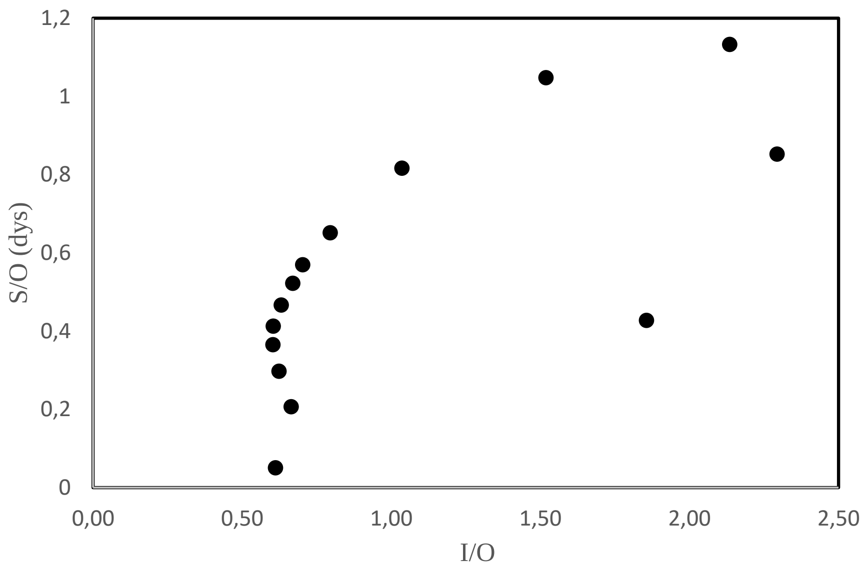


Figure 6. Relationship of I/O versus S/O.

From Figure 6, table 2 and the previous equations given in section 5, we find that the values of m, b, x and K are 0.6, 0.15, 0.8 and 0.75 days respectively. However, it can be noticed that these values are neither realistic nor reasonable to be accepted, for instance the value of x is higher than the actual value. Therefore, we recommend not to use this method. Table 3 gives a comparison of the different methods.

Table 3. Parameter estimation and comparative statistics.

Method	x	Accuracy (%)	K (days)	Accuracy (%)
Classical procedure	0.20	--	1	--
One	0.16	80	0.9	90
Two	0.19	95	1	100
Three	0.80	120	0.75	75

7 CONCLUSION

There are several mathematical procedures that deal with hydrologic flood routing. The Muskingum technique is one of the most common techniques used for flood routing for river reach. In this research three deferent producers are produced and explained in using the Muskingum method in flood routing

The accuracy of method one presented in this research, with respect to the classical procedure is about 80% for x value and about 90% for the travel time parameter K value. The accuracy of method two with respect to the classical procedure is about 95% for x value and about 100% for the K value. The accuracy of method three with respect to the classical procedure is about 120% for x value and about 75% for the K value. Moreover, the accuracy of the methods depends primary on the accuracy of plotting the hydrographs and the accuracy of the ability of the person in reading the values from these hydrographs. The practical advantages of these methods compared to the "classical" ones, when worse results are obtained, are that: those methods do not require storage calculation since they deal directly with the plotted hydrographs. In addition, the methods do not deal with trial-and-error procedure, which needs long calculations and time. Moreover, the methods determine the value of x more accurately than the classical procedure, except for method three, because they assume a value for x and this value may not converge to the exact value. Method three does not give accurate or realistic values for x and K. Therefore, it is recommended not to be used.

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