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CALCULATING A BEAM OF VARIABLE SECTION LYING ON AN ELASTIC FOUNDATION

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In this article, there has been studied the bending state of a reinforced concrete beam with a variable cross-sectional height along its length that rests along its entire length on a brick wall. The beam is under the action of arbitrarily located concentrated forces. The study has been performed on the basis of the original inhomogeneous differential equation of the 4th order, taking into account the external load and the bedding coefficients of the elastic foundation. Using the finite difference method, typical resolving finite difference equations have been obtained. A study of the influence of the degree of elasticity of the base, with a change in the value of the bed coefficients and elasticity parameters, was conducted. The results confirm the reliability of theoretical and practical calculations. Given theoretical provisions and applied results can be used in scientific research in the field of mechanics of a deformable solid body, as well as in practical design.

Keywords: elastic foundation, bending state, reinforced concrete beams

1 INTRODUCTION

It is known that the calculation of beams on an elastic foundation is always a very laborious task and requires cumbersome calculations. In this regard, it becomes necessary to introduce a numerical calculation method based on the finite difference method or the grid method.

In this work, there has been studied the stress-strain state (SSS) of a complex structure in its interaction with an elastic foundation (soil).

This problem is relevant in the field of mechanics of a deformable solid body: for a number of years, many scientists have been involved in it. Articles [1–4] consider constructions on an elastic foundation. The bending and stability of these structures are investigated. The output solutions are performed by analytical and numerical methods.

For example, at works [5, 6] to analyze the buckling of beams (beams of the Timoshenko type) resting on an elastic Winkler foundation, their finite element model was used with the power series of the components of the required displacements; the impact of boundary conditions, the material gradient, the Winkler parameter was studied in detail; at this, it was found that the elastic base improved the stability characteristics.

To study the transverse bending moment under an eccentric vertical load, the shear increment of individual elements of a box-shaped trapezoid profile truss was obtained based on the beam operation on an elastic foundation; the obtained results were compared with the finite element method in the article [7].

In work [8], for high beams with holes, a nonlinear method of analyzing concrete structures is proposed, which makes it possible to predict the presence of cracks, failure modes, and ultimate strengths.

The proposed theoretical provisions can be applied to similar structures operating on an elastic foundation.

Also in [9], the finite difference method was used to study the operation of beams on an elastic foundation under the assumption of a parabolic change in the distribution of contact pressure with an adjacent elastic medium; there was proposed a method of constructing lines of impact of bending moments in beams on an elastic foundation on the action of an arbitrary external load.

For studying continuous beams on elastic supports (bases), finite elements are proposed using the Vlasov foundation model (its modification is proposed), taking into account the effect of shear deformation; the foundation model is a two-parameter elastic medium. There are determined the leading parameters of the calculation are based on the phenomenon of the cross-section rotation when bending beams [10].

Works [11, 12] consider the bending of variable cross-section beams on two-parameter elastic foundations; the method of transferring the matrix of calculated parameters is used; the author's computer program for the calculation is proposed, which makes it possible to automate the main stages of studies and practical calculations to a wide extent.

The analysis of works in the field of studying the SSS of the structures based on elastic foundations (media) shows that today there are quite a lot of unsolved problems and engineering work. In this regard, the theoretical approaches proposed in this work will expand, deepen and clarify the operation of various structures interacting with adjacent elastic media (bases).

2 METHOD AND SOLUTION

In the construction industry, structures often rest along their entire length on the underlying structures or the surface of the earth (soil).

In this paper, there has been studied the bending state of a reinforced concrete beam with a variable cross-sectional height along its length under the action of arbitrarily located concentrated forces on it that rests along its entire length on a brick wall (Fig. 1).

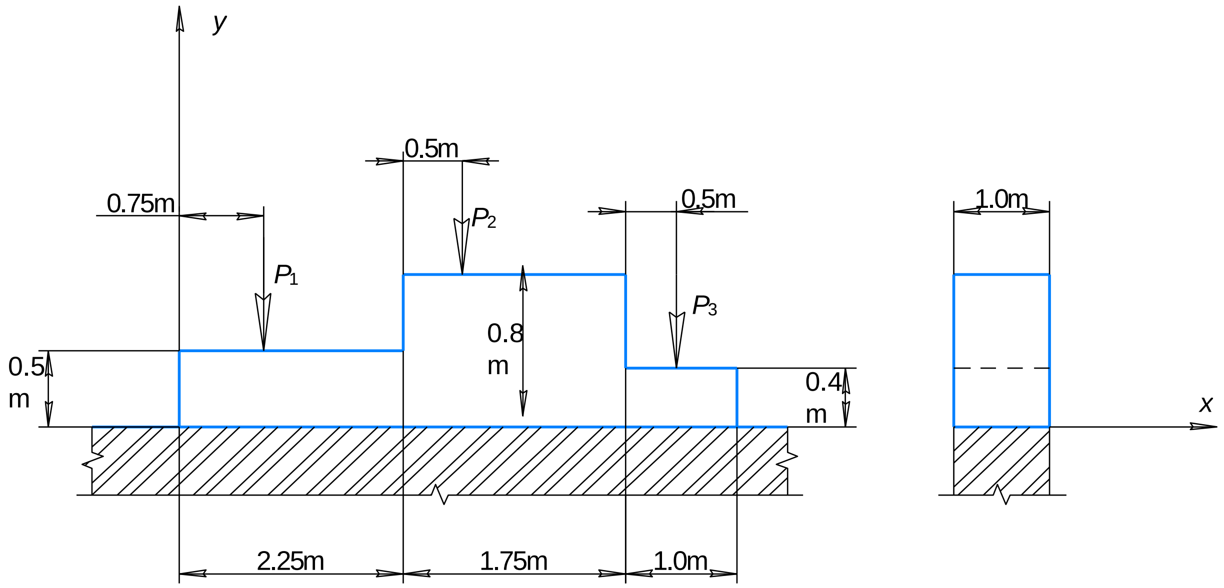


Figure 1: Initial diagram of the structure

The initial data (for three constituent parts) are as follows: inertia moments $I_1 = 10.417 \cdot 10^{-3}$ m^4; $I_2 = 42.667 \cdot 10^{-3}$ m^4; $I_3 = 5.334 \cdot 10^{-3}$ m^4; the elasticity modulus of the reinforced concrete beam $E = 2.1 \cdot 10^4$ MN; the foundation elasticity modulus $E_0 = 0.3 \cdot 10^4$ MN.

The governing differential equation for the bending of a beam lying on an elastic foundation [13, 14]:

$$EI(x)y^{IV} = q(x) - k(x) ,$$

For the beam with variable cross-section the governing equation will take the form:

$$\left[EI(x)y''(x)\right]'' + k(x)y(x) - q(x) = 0 , \tag{1}$$

$y(x)$ is the beam bending function,

$EI(x)$ is the beam bending rigidity,

$k(x)$ is the foundation bedding coefficient function (Winkler constant),

$q(x)$ is the external load distributed along the beam length.

Later on, equation (1) will be realized by the finite difference method (FDM). For the *i*-th linear grid node with equal steps (Fig. 2), equation (1) will have the shape functions [14, 15]:

$$\frac{\partial^4 y}{\partial x^4} = \frac{1}{\lambda^4}\left[6y_i - 4\left(y_k + y_l\right) + \left(y_u + y_v\right)\right],$$

$$\frac{\partial^3 y}{dx^3} = \frac{1}{2\lambda^3}\left[y_u - 2(y_l - y_k) - y_v\right],$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{\lambda^2}\left[y_u - 2y_i + y_v\right],$$

$$\frac{\partial y}{\partial x} = \frac{1}{2\lambda}\left(y_l - y_k\right).$$

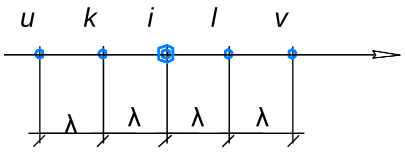


Figure 2: Grid template

Based on these functions, by introducing the rigidity coefficients α_k, α_i, α_l, the resolving finite difference equation is obtained:

$$\alpha_k y_u - 2(\alpha_k + \alpha_i) y_k + \left(\alpha_k + 4\alpha_i + \alpha_l + \frac{k\lambda^4}{EI_0}\right) y_i - 2(\alpha_i + \alpha_l) y_l + \alpha_l y_v = \frac{P_i \lambda^3}{EI_0}, \quad (2)$$

where

$$\alpha_k = \frac{EI_k}{EI_0}, \; \alpha_i = \frac{EI_i}{EI_0}, \; \alpha_l = \frac{EI_l}{EI_0} \quad (3)$$

are the rigidity parameters ($EI_0 = 11.2014 \cdot 10^4$ kNm^2 is the bending rigidity of section 3) (Fig. 3).

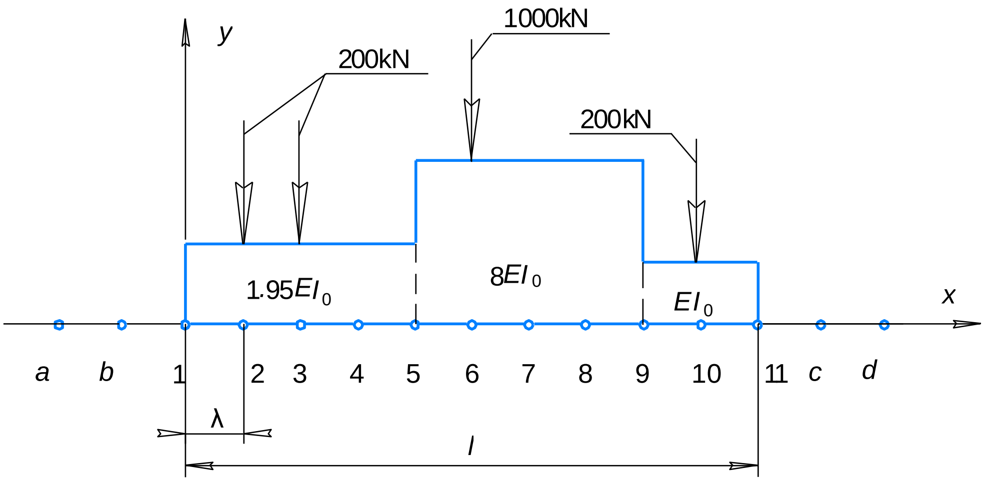


Figure 3: FDM design diagram

A linear grid with a step ($\lambda = l/10 = 5/10 = 0.5$ m) on the lower face of the reinforced concrete beam is applied, (Fig. 3), here: 1, 2, 3, ..., 11 are design (intra-loop nodes), a, b, c, d are contour nodes that will be excluded using the boundary conditions at the ends of the beam (nodes 1, 11).

From the boundary conditions (Fig. 3):

$$M_1 = EI\left(\frac{d^2y}{dx^2}\right)_1 = 0, \text{ from here } y_b = 2y_1 - y_2, \quad (4)$$

$$Q_1 = EI\left(\frac{d^3y}{dx^3}\right)_1 = 0, \text{ from here } y_a = 4(y_1 - y_2 + y_3), \quad (5)$$

$$M_{11} = EI\left(\frac{d^2y}{dx^2}\right)_{11} = 0, \text{ from here } y_b = 2y_{11} - y_{10}, \quad (6)$$

$$Q_{11} = EI\left(\frac{d^3y}{dx^3}\right)_{11} = 0, \text{ from here } y_2 = 4(y_{11} - y_{10}) + y_9 \quad (7)$$

According to Fig. 3:

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 1.953; \; \alpha_6 = \alpha_7 = \alpha_8 = 8.0; \; \alpha_9 = 4.5; \; \alpha_{10} = \alpha_{11} = 1.0. \quad (8)$$

Next the bedding coefficient for a brick wall is determined ($E_0 = 3 \cdot 10^6$ kN/m^2, μ =0.42).

a) According to [16]:

$$C_z = \kappa_z \frac{E_0}{(1-\mu^2)\sqrt{F}} = 3.258 \cdot 10^6 \text{ kN/m}^3, \quad (9)$$

$\kappa_z = 2.0$ (Table 6.1, [16]), $\mu = 0.42$ (Table 1.15, [16]), $F = 1 \times 5 = 5$ m^2 is the area of the reinforced beam foundation base.

b) According to [17]:

$$C_s = \frac{E_s}{f\sqrt{F}}, \tag{10}$$

$E_s = E_0 = 3\cdot 10^6$ kN/m², $f = 0.4$ (Table 6, [17]).

According to (10): $C_s = \dfrac{3\cdot 10^6}{0.4\sqrt{5}} = 3.354\cdot 10^6$ kN/m³.

K is the the average value, that accounting for the base elasticity [14].

$K = C_z = C_s = (3.354 + 3.258)\cdot 10^6 / 2 = 3.306\cdot 10^6$ kN/m³.

According to formulas (9, 10), we can calculate the bedding coefficients and the parameters of the base (soil) elasticity (Fig. 1):

a) For section 1 ($\mu = 0.42$, $F_1 = 2.25$ m², $\lambda = 0.5$ m).

$$C_1 = 2.0\frac{3\cdot 10^6}{(1-0.42^2)\sqrt{2.25}} = 4.8567\cdot 10^6 \text{ kN/m}^3,\ K_1 = \frac{4.8567\cdot 10^6(0.0625)}{11.2014\cdot 10^4} = 2.7099;$$

b) For section 2 ($F_2 = 1.75$ m²):

$$C_2 = 2.0\frac{3\cdot 10^6}{0.8236\cdot\sqrt{1.75}} = 5.5066\cdot 10^6 \text{ kN/m}^3,\ K_2 = \frac{5.5066\cdot 10^6(0.0625)}{11.2014\cdot 10^4} = 3.0725;$$

c) For section 3 ($F_3 = 1$ m²):

$$C_3 = 2.0\frac{3\cdot 10^6}{0.8236\sqrt{1}} = 7.285\cdot 10^6 \text{ kN/m}^3, K_3 = \frac{7.285\cdot 10^6\cdot(0.0625)}{11.2014\cdot 10^4} = 4.0678;$$

$$K_1 = 2.7099;\ K_2 = 3.0725;\ K_3 = 4.0678. \tag{11}$$

To design the beam with variable bending rigidity by the finite differences method (FDM), the values of the K_1, K_2, K_3 coefficients are used (Fig. 3). Table 1 gives the resolving square matrix of the 11th order that corresponds to the FGM:

$$A\cdot\vec{y} = \vec{P} \tag{12}$$

$\vec{y} = \left|\{y_1, y_2, \ldots, y_{12}\}\right|$ is the vector of the nodal displacements (deflections) of the beam;

$\vec{P} = \left|\{P_1, P_2, \ldots, P_{12}\}\right|$ is the free member vector that account for the external load, where $y_i = \dfrac{P_i\lambda^3}{EI_0}$ (P_i is the nodal load, Fig. 3).

$$\begin{bmatrix}
1.953\cdot 2 + C_1 & -3.906\cdot 2 & 1.953\cdot 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-3.906 & 9.765 + C_1 & -7.812 & 1.953 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.953 & -7.812 & 11.718 + C_1 & -7.812 & 1.953 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.953 & -7.812 & 11.718 + C_1 & -7.812 & 1.953 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.953 & -7.812 & 17.765 + C_1 & -19.906 & 8 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.953 & -19.906 & 41.95 + C_2 & -32 & 8 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 8 & -32 & 48 + C_2 & -32 & 8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 8 & -32 & 44.5 + C_2 & -25 & 4.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 8 & -25 & 27 + 0.5(C_1 + C_2) & -11 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.5 & -11 & 8.5 + C_3 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & -4 & 2 + C_3
\end{bmatrix}
\begin{pmatrix}
0 \\ 2.232\cdot 10^{-4} \\ 2.232\cdot 10^{-4} \\ 0 \\ 0 \\ 11.16\cdot 10^{-4} \\ 0 \\ 0 \\ 0 \\ 2.232\cdot 10^{-4} \\ 0
\end{pmatrix}$$

Figure 4: Resolving FGM matrix (in equation 12)

Solving equation 12 and obtaining the $\vec{y}$ vector (Table 1)

Table 1: Beam deflections at the grid nodes (Fig. 3)

No.	1	2	3	4	5	6	7	8	9	10	11
y_i, m	$2.96\cdot 10^{-5}$	$5.72\cdot 10^{-5}$	$6.42\cdot 10^{-5}$	$6.52\cdot 10^{-5}$	$9.98\cdot 10^{-4}$	$1.17\cdot 10^{-4}$	$8.42\cdot 10^{-5}$	$4.90\cdot 10^{-5}$	$2.77\cdot 10^{-5}$	$2.87\cdot 10^{-5}$	$7.80\cdot 10^{-6}$

The value of the s response of the base (soil) is calculated by the formula:

$$P(x) = C(x) \cdot y(x) \quad (13)$$

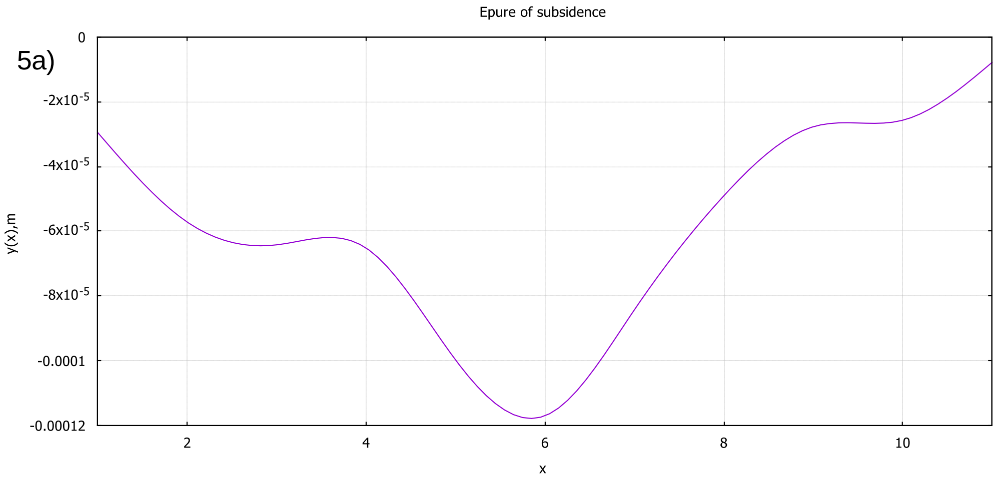
$C(x)$ is the coefficient (parameter) of the base bedding elasticity (expression 11).

The values of bending moments at the grid nodes are calculated by the formula (Fig. 2):

$$M_i = EI_i \left(d^2y / dx^2\right) = \frac{EI_i}{\lambda^2}\left(y_k - 2y_i + y_l\right) \quad (14)$$

Figure 5 shows the results of designing the beam (Fig. 3) in the form of deflection curves y_i (Fig. 5a); the curves of the base (soil) response and bending moments (Fig. 5b).

5a)



5b)

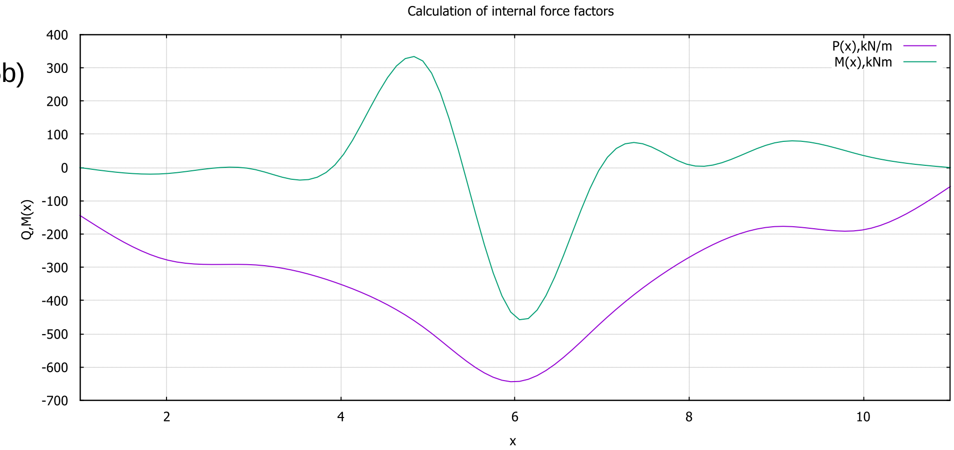


Figure 5: The results of designing the beam in Fig. 1

In work [18], the beam shown in Fig. 1 was designed using the method of Professor B.N. Zhemochkin. The following results have been obtained:

$P_6(x) = 494$ kN/m, (15)

$M_6(x) = 271$ kNm.

Values (15) are close to the values shown in Fig. 3b, c.

The study of the impact of the degree of the base elasticity was performed by changing the values of the bedding coefficients (K_i) and elasticity parameters (C_i) (Table 2).

Table 2: Bedding coefficients and the foundation elasticity parameters

No.	Elasticity modulus of the base, kPa	elasticity parameters (C_i), kN/m^3			bedding coefficients (K_i)		
		C_1	C_2	C_3	K_1	K_2	K_3
1	$3.0 \cdot 10^6$	$4.8767 \cdot 10^6$	$5.5066 \cdot 10^6$	$7.285 \cdot 10^6$	2.7099	3.0725	4.06478
2	$3.0 \cdot 10^5$	$4.8567 \cdot 10^5$	$5.5066 \cdot 10^5$	$7.285 \cdot 10^5$	0.2710	0.3073	0.4065
3	$1.5 \cdot 10^5$	$4.8567 \cdot 10^4$	$5.5066 \cdot 10^4$	$7.285 \cdot 10^4$	0.136	0.154	0.203

No.	Elasticity modulus of the base, kPa	elasticity parameters (C_i), kN/m^3			bedding coefficients (K_i)		
		C_1	C_2	C_3	K_1	K_2	K_3
4	$1.0 \cdot 10^5$	$4.8567 \cdot 10^3$	$5.5066 \cdot 10^3$	$7.285 \cdot 10^3$	0.09	0.102	0.135
5	$0.75 \cdot 10^4$	$4.8567 \cdot 10^2$	$5.5066 \cdot 10^2$	$7.285 \cdot 10^2$	0.06	0.077	0.102
6	$0.6 \cdot 10^4$	48.567	55.066	72.85	0.05	0.061	0.081
7	$5.0 \cdot 10^4$	4.8567	5.5066	7.285	0.04	0.051	0.058
8	$4.2 \cdot 10^3$	4.8567	5.5066	7.285	0.034	0.0844	0.051

According to Table 2, there has been designed a beam of variable bending rigidity located on an elastic foundation (soil) (Fig. 1). Fig. 6 shows the results on the deflections at nodes 1, 3, 6, 10, 11 of the beam (Fig. 3) dependence on the change in the values of the elasticity modulus of the base material (soil) with their monotonous decrease by one order.

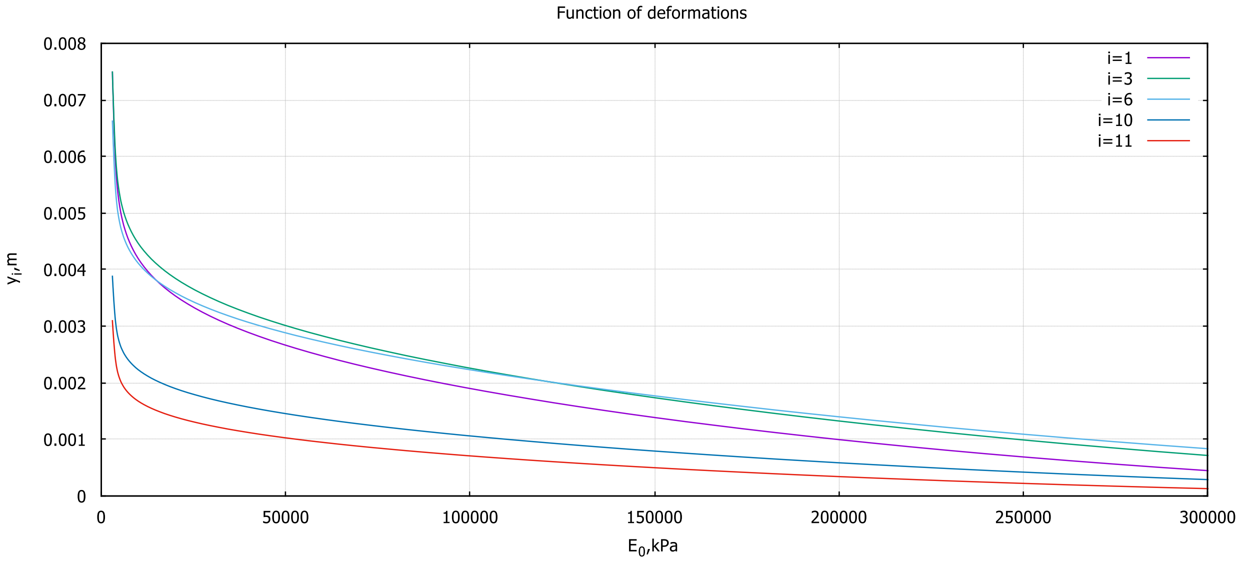


Figure 6. Dependence of a beam lying on an elastic foundation deflection on the values of the foundation material deformation modulus

3 CONCLUSIONS

1. Based on the resolving matrices of the FDM, a test calculation of the beam shown in Fig. 3 that lies on a brick base; deflections in the beam nodes (Fig. 3) are obtained, shown in Table 1; according to formulas (13, 14), the values of support reactions (repulse of the base) and bending moments along the axis of the reinforced concrete beam have been determined.
2. A study of the influence of the properties of an elastic foundation (soils) on the magnitude of deflections at the grid nodes was conducted. It has been determined that with a monotonous increase in the value of E_0, the deflections at the beam nodes decrease monotonically at the nodes 1, 3, 6, 10, 11 of the beam.
3. Obtained in this paper theoretical and practical results will widely use as in construction practice, so in scientific research of building structures on elastic foundations.

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